

# Entropy & Second Law Cheat Sheet

Entropy balance, T-ds relations, ideal-gas entropy and isentropic efficiencies

**ENGINEERING**  
**CHEAT SHEET**

## Key equations

| Quantity                                    | Equation                                                                                                                                            | Notes                                                               |
|---------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|
| <b>Clausius inequality</b>                  | $\oint \delta Q/T \leq 0$                                                                                                                           | Equality only for a reversible cycle                                |
| <b>Entropy change</b>                       | $\Delta S = S_2 - S_1 = \int \delta Q_{\text{rev}}/T$                                                                                               | Units J/K; specific entropy $s$ in J/(kg·K)                         |
| <b>T-ds relations</b>                       | $T ds = du + P dv$ $T ds = dh - v dP$                                                                                                               | Basis for entropy of any simple compressible substance              |
| <b>Ideal gas, constant <math>c_p</math></b> | $s_2 - s_1 = c_p \ln(T_2/T_1) - R \ln(P_2/P_1)$                                                                                                     | Variable $c_p$ : use $s^\circ$ tables                               |
| <b>Ideal gas, constant <math>c_v</math></b> | $s_2 - s_1 = c_v \ln(T_2/T_1) + R \ln(v_2/v_1)$                                                                                                     |                                                                     |
| <b>Incompressible substance</b>             | $s_2 - s_1 = c \ln(T_2/T_1)$                                                                                                                        | Solids and liquids                                                  |
| <b>Entropy balance (control volume)</b>     | $dS_{\text{cv}}/dt = \sum \dot{Q}_j/T_j + \sum \dot{m}_{\text{in}} s_{\text{in}} - \sum \dot{m}_{\text{out}} s_{\text{out}} + \dot{S}_{\text{gen}}$ | $\dot{S}_{\text{gen}} \geq 0$ ; entropy is generated, not conserved |
| <b>Isentropic ideal-gas process</b>         | $T_2/T_1 = (P_2/P_1)^{(k-1)/k}$                                                                                                                     | Constant $c_p$ , $k = c_p/c_v$                                      |
| <b>Turbine efficiency</b>                   | $\eta_t = \text{actual work out} / \text{isentropic work out}$                                                                                      | Exit state 2s: $s_{2s} = s_1$ , $P_{2s} = P_2$                      |
| <b>Compressor efficiency</b>                | $\eta_c = \text{isentropic work in} / \text{actual work in}$                                                                                        |                                                                     |
| <b>Nozzle efficiency</b>                    | $\eta_n = \text{actual } \Delta KE / \text{isentropic } \Delta KE$                                                                                  |                                                                     |

## Key takeaways

- Kelvin–Planck: no cyclic engine can exchange heat with only one reservoir and produce net work. Clausius: heat will not flow cold to hot without external work. The two statements are equivalent.
- No engine beats a Carnot engine between the same reservoirs, and all reversible engines between them are equally efficient.
- A reversible refrigerator or heat pump has a higher COP than an irreversible one between the same reservoirs.
- Sources of irreversibility: finite  $\Delta T$  heat transfer, friction, unrestrained expansion, mixing, chemical-potential differences, hysteresis.
- Isentropic = adiabatic *and* reversible ( $\Delta s = 0$ ). It is the ideal benchmark for turbines, compressors and nozzles.
- Property tables give  $s$  for real fluids like steam; the ideal-gas formulas above are for gases and incompressible solids/liquids use the  $c \ln(T)$  form.