Dynamic Systems Modeling Cheat Sheet

Linear vs nonlinear, linearization, state space

Key equations

Quantity	Equation	Notes
Straight-line model	$y = m x + b$	Article's simple linear example
Superposition (linearity)	$\alpha u_1 + \beta u_2 \rightarrow \alpha y_1 + \beta y_2$	If $u_1 \rightarrow y_1$ and $u_2 \rightarrow y_2$
Nonlinear state model	$\dot{x} = f(x, u), y = g(x, u)$	Typical form; equations in the article are images
Linearization	$\Delta\dot{x} = A \Delta x + B \Delta u$, $A = \partial f/\partial x$, $B = \partial f/\partial u$ at (x_0, u_0)	First-order Taylor expansion about the operating point
State space (LTI)	$\dot{x} = A x + B u, y = C x + D u$	Realizations are not unique
Transfer function	$G(s) = Y(s)/U(s) = C (sI - A)^{-1} B + D$	Frequency-domain input–output relation
Discrete time	$x[k+1] = A_d x[k] + B_d u[k]$	Difference equations vs continuous differential equations

Key takeaways

- Static models relate variables at one instant; dynamic models describe how they evolve and so can predict behaviour and stability.
- Linear models obey superposition and homogeneity. Linear time-invariant (LTI) systems have lumped components and constant parameters.
- Linear time-varying example from the article: a vehicle or rocket whose mass falls as fuel is burned.
- Most physical systems are nonlinear, but a nonlinear system is well approximated by a linear one in a small neighbourhood of an operating point.
- Transfer functions are generally linear and say nothing about internal workings; the same transfer function can describe different systems.
- State variables are the minimum set that uniquely defines the system state; state-space handles multi-input, multi-output cases that classical frequency-domain methods struggle with.
- Worked examples in the article: pendulum (linearization) and a car suspension model. Their equations are images, so the general forms above are standard textbook ones.