Lecture 26b:
Summary of Availability/Exergy

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Updated material and slides from

Review Availability and Exergy

- Mass balance for CV: $$\frac{dm_{CV}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}$$
  - For CM: m=constant
- Energy balance for CV:

$$\frac{dE_{CV}}{dt} = \dot{Q} + \dot{W} + \sum_{in} \dot{m}_{in}\left(h + \frac{v^2}{2} + gz\right)_{in} - \sum_{out} \dot{m}_{out}\left(h + \frac{v^2}{2} + gz\right)_{out}$$

  - For CM: $dE_{CM} = dU_{CM} + dKE_{CM} + dPE_{CM} = \dot{Q}dt + \dot{W}dt$
- Entropy Balance for CV:

$$\frac{dS_{CV}}{dt} = \sum_{j} \frac{\dot{Q}_j}{T_{b,j}} + \sum_{in} \dot{m}_{in}s_{in} - \sum_{out} \dot{m}_{out}s_{out} + \dot{\sigma}$$

  - For CM: $$\frac{dS_{CM}}{dt} = \sum_{i} \frac{\dot{Q}_i}{T_{b,i}} + \dot{\sigma}$$

Review Availability and Exergy

- Availability or non-flow exergy for closed systems:

$$\Phi = \mathrm{m}\phi = m[(\mathrm{u}-\mathrm{u}_0)+\mathrm{P}_0(\mathrm{v}-\mathrm{v}_0)-\mathrm{T}_0(s-s_0)+\mathrm{ke}+\mathrm{pe}]$$

- Relationship between useful work and availability:

$$W_{act,use,tot} = \left(E_2 - E_1\right) + p_o\left(V_2 - V_1\right) - T_o\left(S_2 - S_1\right) - \sum_j Q_j\left(1 - \frac{T_o}{T_j}\right) + T_o\sigma_{tot}$$

$$\Rightarrow \quad W_{act,use,tot} = \Phi_2 - \Phi_1 - \sum_j Q_j\left(1 - \frac{T_o}{T_j}\right) + T_o\sigma_{tot}$$

T_j = temp. of reservoir

$$\text{or} \quad W_{act,use,CM} = \Phi_2 - \Phi_1 - \sum_i Q_i\left(1 - \frac{T_o}{T_i}\right) + T_o\sigma_{CM}$$

T_i = temp. of sys. bound.

Review Availability and Exergy

- Stream availability or flow exergy:

$$\dot{\Psi} = \dot{\mathrm{m}}\psi = \dot{\mathrm{m}}\left[(\mathrm{h}-\mathrm{h_o}) + \frac{\mathrm{v}^2}{2} + \mathrm{g}(\mathrm{z}-\mathrm{z_o}) - \mathrm{T_o}(\mathrm{s}-\mathrm{s_o})\right]$$

- Exergy balance for CV:

$$\frac{\mathrm{d}\Phi_{CV}}{\mathrm{dt}} = \sum_{in} \dot{m}_{in}\,\psi_{in} - \sum_{out} \dot{m}_{out}\,\psi_{out} + \sum_{j} \dot{Q}_j\left(1-\frac{T_o}{T_j}\right) + \left(\dot{W}_{\mathrm{CV}} + p_o\frac{dV_{CV}}{dt}\right) - T_o\dot{\sigma}_{CV}$$

$$\dot{\mathrm{W}}_{\mathrm{CV}} = \dot{\mathrm{W}}_{\mathrm{use}} - \mathrm{p_o}\frac{\mathrm{dV_{CV}}}{\mathrm{dt}}$$

- Irreversibilities or exergy destruction $\dot{I}_{CV} = T_o\dot{\sigma}_{CV}$

Review Availability and Exergy

- Heat Engine (power cycle):
  - » Thermal efficiency (based on First Law):

$$\eta_{\text{thermal,HE}} = \frac{|W_{net}|}{Q_H} = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{Q_H}$$

  - » Reversible heat engine (Carnot heat engine)

$$\eta_{\text{thermal,HE,rev}} = \eta_{\text{Carnot,HE}} = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \frac{T_L|\Delta S_L|}{T_H|\Delta S_H|} = 1 - \frac{T_L}{T_H}$$

  - » Second Law efficiency/effectiveness
    - – For reservoirs:

$$\varepsilon_{\text{II,HE}} = \eta_{\text{II,HE}} = \frac{\eta_{\text{thermal,HE}}}{\eta_{\text{Carnot,HE}}} = \frac{\left|\dot{W}_{out,use}\right|}{\dot{\Phi}_{Q_H}} = \frac{\eta_{th,act}\left|\dot{Q}_H\right|}{\left|\dot{Q}_H\right| \underbrace{\left(1 - \frac{T_0}{T_H}\right)}_{=\eta_{th,Carnot}}}$$

    - – Finite temperatures:

$$\varepsilon_{\text{II,HE}} = \eta_{\text{II,HE}} = \frac{\left|\dot{W}_{out,use}\right|}{\Delta\dot{\Psi}_H}$$

Review Availability and Exergy

- Heat Pump (Air-conditioning/Refrigeration):
  - » Coefficient of Performance (cooling mode C):

$$COP_{C,act} = \frac{|\dot{Q}_L|}{\dot{W}}$$

  - » Reversible heat pump cycle (cooling mode)

$$COP_{C,Carnot} = \frac{T_L}{T_H - T_L}$$

  - » Second Law efficiency/effectiveness
    - – For reservoirs:

$$\varepsilon_{HP,C} = \frac{COP_{C,act}}{COP_{C,Carnot}} = \frac{\dot{\Phi}_{Q_L}}{\dot{W}}$$

    - – General expression:

$$\varepsilon_{\mathrm{HP,C}} = \frac{COP_{C,act}}{COP_{C,Carnot}}$$

Review Availability and Exergy

- Heat Pump (Heating):
  - » Coefficient of Performance (heating mode H):

$$COP_{H,act} = \frac{|\dot{Q}_H|}{\dot{W}}$$

  - » Reversible heat pump cycle (heating mode)

$$COP_{H,Carnot} = \frac{T_H}{T_H - T_L}$$

  - » Second Law efficiency/effectiveness
    - – For reservoirs:

$$\varepsilon_{HP,H} = \frac{COP_{H,act}}{COP_{H,Carnot}} = \frac{\dot{\Phi}_{Q_H}}{\dot{W}}$$

    - – General expression:

$$\varepsilon_{\mathrm{HP,H}} = \frac{COP_{H,act}}{COP_{H,Carnot}}$$

Review Availability and Exergy

- Turbine (adiabatic):
  - » Isentropic efficiency: $$\eta_{\text{t,is}} = \frac{|w_{\text{actual}}|}{|w_{\text{isentropic}}|}$$
  - » Exergetic efficiency/effectiveness $$\varepsilon_t = \frac{|\dot{W}_{\text{actual}}|/\dot{m}}{\psi_{in} - \psi_{out}}$$
- Compressor/Pump (adiabatic):
  - » Isentropic efficiency: $$\eta_{\text{c,is}} = \frac{w_{\text{isentropic}}}{w_{\text{actual}}}$$
  - » Exergetic efficiency/effectiveness: $$\varepsilon_t = \frac{\psi_{out} - \psi_{in}}{\dot{W}_{\text{actual}}/\dot{m}}$$

Review Availability and Exergy

- Heat exchanger without mixing and adiabatic (c=cold stream; h=hot stream):
  - » Exergetic efficiency/effectiveness

$$\varepsilon_{HX} = \frac{\dot{m}_{\mathrm{c}}(\psi_{c,out} - \psi_{c,in})}{\dot{m}_{\mathrm{h}}(\psi_{h,in} - \psi_{h,out})}$$

- Direct contact heat exchanger (mixing) adiabatic:
  - » Hot stream: 1
  - » Cold stream: 2
  - » Mixed stream: 3

$$\varepsilon_{mix} = \frac{\dot{m}_2(\psi_3 - \psi_2)}{\dot{m}_1(\psi_1 - \psi_3)}$$

- General definition of second law efficiency/effectiveness:

$$\varepsilon_{II} = \frac{\text{Exergy recovered}}{\text{Exergy supplied}} = 1 - \frac{\text{Exergy destroyed}}{\text{Exergy supplied}}$$

