ME200 Course Review
Fall 2020

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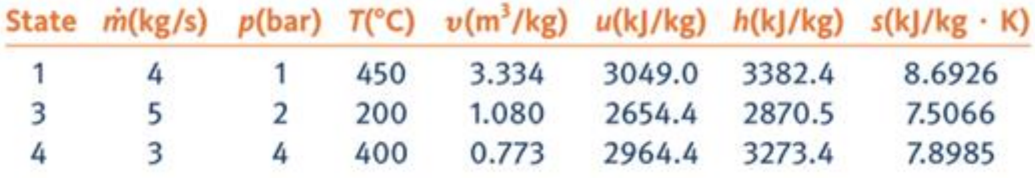
Friday, 12/4/2020 & Sunday 12/6/2020

Acknowledgements: R. Barta, A. Nash, S. Naik

Some tips for Final Exam

1. For problems: EFD, Basic Equations Assumptions, calculate/retrieve known properties
2. It is highly recommended to create a state property summary table especially for cycles:



State	$\dot{m}$(kg/s)	p(bar)	T(°C)	v(m^3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg · K)
1	4	1	450	3.334	3049.0	3382.4	8.6926
3	5	2	200	1.080	2654.4	2870.5	7.5066
4	3	4	400	0.773	2964.4	3273.4	7.8985

3. Review Equation Sheet
4. SI (or kSI) units; avoid “bar”
   - Mandatory for Ideal Gas Law, 2nd law calculations

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BASIC EQUATIONS FOR ME 200

Mass Conservation, 1st Law and 2nd Law Relations

$$\left.\frac{dm}{dt}\right|_{system} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}$$

$$\left.\frac{dE}{dt}\right|_{system} = \sum_{in} \dot{m}_{in}\,(h + ke + pe)_{in} - \sum_{out} \dot{m}_{out}\,(h + ke + pe)_{out} + \dot{Q} - \dot{W}$$

$$\left.\frac{dS}{dt}\right|_{system} = \sum_{in} \dot{m}_{in}\, s_{in} - \sum_{out} \dot{m}_{out}\, s_{out} + \sum_{j} \frac{\dot{Q}_j}{T_{j,boundary}} + \dot{\sigma}_{generation}$$

Work and Heat Transfer Relations

$$W_{boundary} = \int p\,dV \qquad W_{electrical} = \int \varepsilon\, i\, dt \qquad W_{spring} = \int k_{spring}\, x\, dx \qquad W_{rot} = \int \tau\, d\theta$$

$$w_{int,rev,flow} = -\int_1^2 v\,dp + (V_1^2 - V_2^2)/2 + g(z_1 - z_2) \qquad q_{int,rev} = \int T\,ds$$

PROPERTY RELATIONS FOR ME 200

General Relations

$$h = u + pv \qquad c_v = \left(\frac{\partial u}{\partial T}\right)_v \qquad c_p = \left(\frac{\partial h}{\partial T}\right)_p \qquad E = m(u + ke + pe), ke = V^2/2\,, pe = gZ$$

$$Tds = du + pdv \qquad Tds = dh - vdp \qquad z_{mixture} = x\, z_g + (1 - x) z_f, z = property$$

Models

$$pV = mRT \qquad R = \bar{R}/M \qquad R = c_p - c_v \qquad k = \frac{c_p}{c_v}$$

$$du = c_v dT \quad dh = c_p dT \qquad ds = c_v \frac{dT}{T} + R\frac{dv}{v} \qquad ds = c_p \frac{dT}{T} - R\frac{dp}{p} \qquad s_2 - s_1 = s_2^o - s_1^o - R\ln(\frac{p_2}{p_1})$$

$$\left(\frac{p_2}{p_1}\right)_s = \left(\frac{p_{r2}}{p_{r1}}\right)_s \qquad \left(\frac{v_2}{v_1}\right)_s = \left(\frac{v_{r2}}{v_{r1}}\right)_s \qquad \left(\frac{T_2}{T_1}\right)_s = \left(\frac{p_2}{p_1}\right)_s^{(k-1)/k} \qquad \left(\frac{T_2}{T_1}\right)_s = \left(\frac{v_1}{v_2}\right)_s^{k-1}$$

$$du = cdT \qquad dh = cdT + vdP \qquad ds = c\frac{dT}{T} \qquad h_{comp\ liq}(p, T) = h_f(T) + v_f(T)[p - p_{sat}(T)]$$

EFFICIENCY AND CYCLE RELATIONS FOR ME 200

$$\eta_{turbine} = \frac{w_{act}}{w_s} \qquad \eta_{compressor} = \eta_{pump} = \frac{w_s}{w_{act}} \qquad \eta_{HX} = \frac{(h_{H,in} - h_{H,out})}{(h_{H,in} - h_{C,in})} \qquad \eta_{nozzle} = \frac{V_{2,act}^2}{V_{2,s}^2}$$

$$\eta_{th} = \frac{W_{net,out}}{Q_H} \qquad COP_R = \frac{Q_C}{W_{net,in}} \qquad COP_{HP} = \frac{Q_H}{W_{net,in}} \qquad mep = \frac{W_{net}}{v_{max} - v_{min}} \qquad bwr = \frac{|w_c|}{w_t}$$

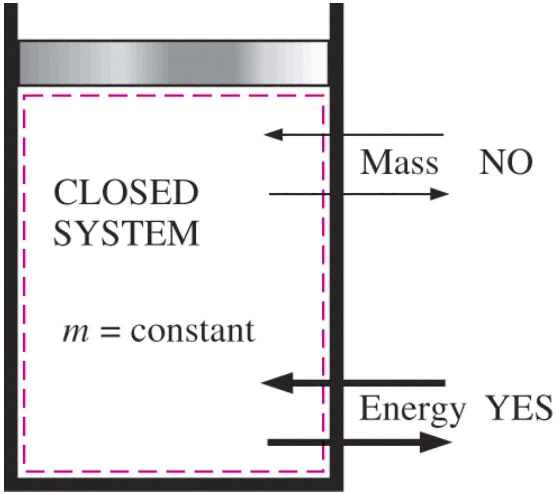
$$\eta_{th,rev} = 1 - \frac{T_C}{T_H} \qquad COP_{R,rev} = \frac{T_C}{T_H - T_C} \qquad COP_{HP,rev} = \frac{T_H}{T_H - T_C} \qquad \left(\frac{T_C}{T_H}\right)_{rev} = \left(\frac{Q_C}{Q_H}\right)_{rev}$$

Thermo in a Nutshell

1. There is a game. (0th law, thermal equilibrium)
2. The best you can do is tie. (1st Law, energy balance)
3. You can never tie. (2nd Law, entropy increase principal)

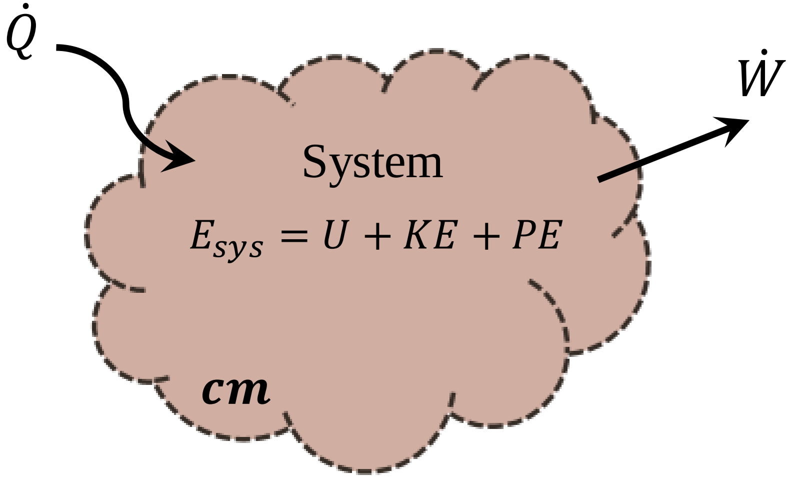
Closed systems (control mass)

- A system in which the mass within the boundary remains constant.
- No mass transfer occurs across the system boundary
- **Heat and work interactions across the boundary ARE still possible**



Here we analyze a specific amount, or piece, of mass

Conservation of energy for a closed system (control mass, *cm*)



1st Law of Thermodynamics
(Conservation of Energy)
for a closed system

$$\Delta E_{sys} = \Delta KE_{sys} + \Delta PE_{sys} + \Delta U_{sys}$$
$$= KE_2 - KE_1 + PE_2 - PE_1 + U_2 - U_1$$

$$\begin{bmatrix}\text{Time rate of change}\\ \text{of the energy}\\ \text{contained within}\\ \text{the system}\\ \text{at time t}\end{bmatrix} = \begin{bmatrix}\text{net rate at which}\\ \text{energy is being}\\ \text{transferred in}\\ \text{by heat at}\\ \text{time t}\end{bmatrix} - \begin{bmatrix}\text{net rate at which}\\ \text{energy is being}\\ \text{transferred out}\\ \text{by work at}\\ \text{time at}\end{bmatrix}$$

$$\boxed{\Delta E_{sys} = E_2 - E_1 = Q - W}$$

$$\uparrow$$

$$\frac{dE_{cm}}{dt} = \dot{Q} - \dot{W} \longrightarrow \int_1^2 \left(\frac{dE_{cm}}{dt}\right) dt = \int_1^2 \dot{Q}\, dt - \int_1^2 \dot{W}\, dt$$

Mechanical forms of work

- Acceleration work (kinetic energy)
- Work to raise a body in a potential field (potential energy)
- Spring work $W_{spr} = \int_{x_1}^{x_2} k_{spring} x dx$
- Shaft work $W_{rot} = \int_{\theta_1}^{\theta_2} \tau d\theta$
- Electrical work $\dot{W}_{elec} = VI$, assuming constant power $W_{elec} = VI \cdot \Delta t$

Characteristics of work

- Work is done by a system on its surrounding if the sole effect on everything external to the system could have been the raising of a weight

- Some important characteristics
  - Work enters or leaves the system through the boundaries
  - Work is not a property
  - Work is associated with a process (change of states)
  - Work is a function of the path followed throughout the process.

Sign Convention:

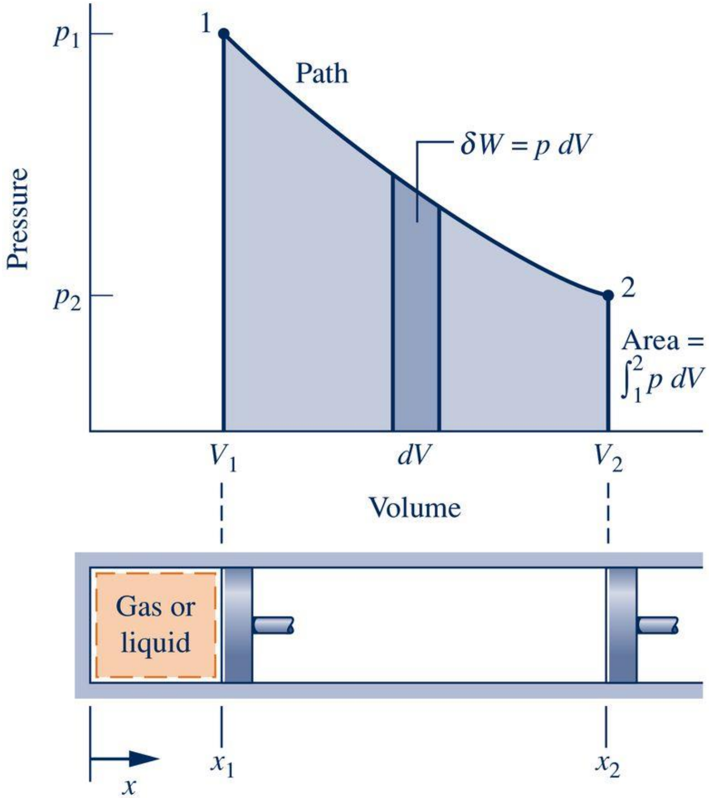
W > 0 → work done **by** the system
(work output)
W < 0 → work done **on** the system
(work input)

Moving boundary work

- As system expands and contracts, the boundary moves.
- Work interactions between the system and surroundings occur when the volume of a fluid changes

$$\boxed{W = \int_{V} P\,dV}$$

Integral represents area under a P-v diagram



Polytropic process

- *Polytropic processes* are described by the following relationship for pressure and volume.

$pv^n = Constant$ $p\cancel{V}^n = Constant$ n: polytropic coefficient

Polytropic process cases	
$n \neq 1$	$W = \frac{C\cancel{V}_2^{1-n} - C\cancel{V}_1^{1-n}}{1-n}, C = P_1\cancel{V}_1^n = P_2\cancel{V}_2^n$
$n = 1$	$W = C\ln\left(\frac{\cancel{V}_2}{\cancel{V}_1}\right), C = P_1\cancel{V}_1 = P_2\cancel{V}_2$
$n = 0$	$W = P(V_2 - V_1), P = P_1 = P_2$

Heat transfer

- Energy transfer between a system and its surroundings that is not associated with mass transfer or work.
- Driven by a temperature difference between a system and its surroundings.
- Like work , heat transfer is a **path function**; NOT a property

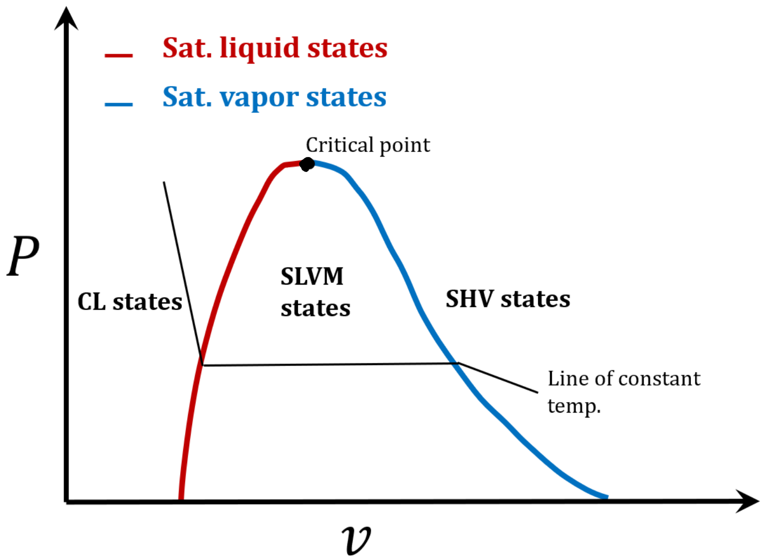
Sign Convention:

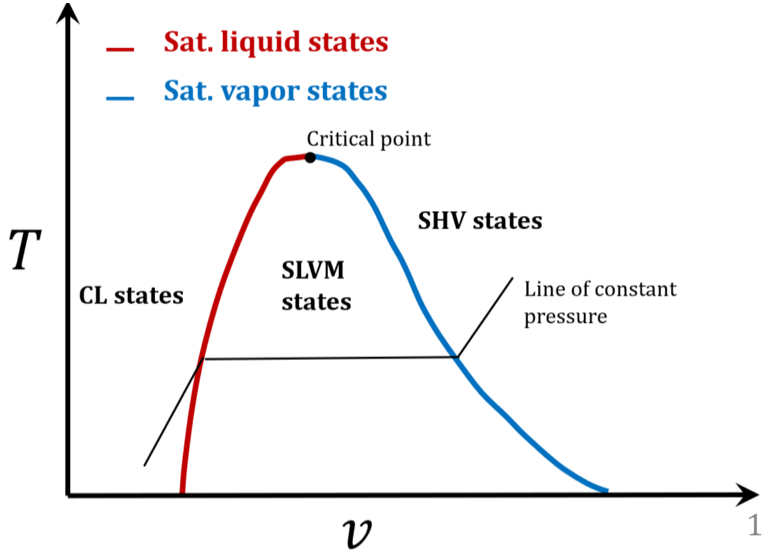
Q > 0 → heat **into** the system (heat input)
Q < 0 → heat **leaving** the system (heat output)

Summary of phases

- Compressed liquid: to the left of the vapor dome
- Saturated liquid: on the vapor dome, left of the critical point
- Saturated liquid-vapor mixture: under the vapor dome, two-phase region; here P, T are dependent properties, need another property to fix the state
- Saturated vapor: on the vapor dome, right of the critical point
- Superheated vapor: to the right of the vapor dome

- Critical point: point where saturated liquid and saturated vapor co-exist; cannot identify phases above this point
- Saturation temperature: temp. at which phase change takes place for a given pressure
- Saturation pressure: pressure at which phase change takes place for a given temp.





Property evaluation

- For simple compressible substances, two **independent, intensive** properties are required to define a state.
- Once a state is defined **ALL** properties at the state can be determined.

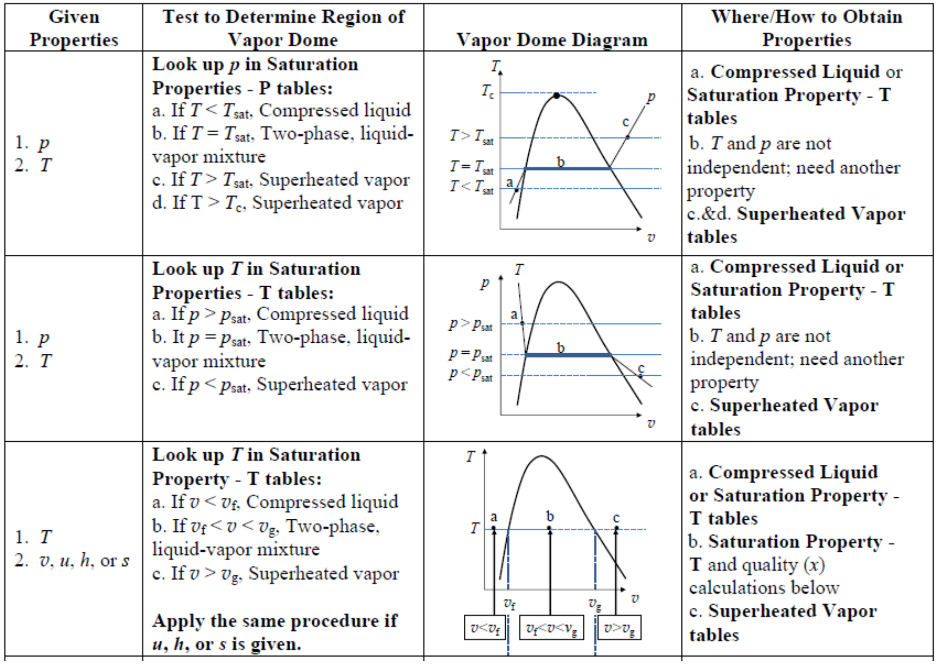
- Properties are listed in the form of tables
- Property information is available for compressed (subcooled) liquid, saturated liquid, liquid-vapor mixture, saturated vapor, and superheated vapor for many substances

- Property tables usually list specific properties
  - Specific volume ($v = V/m$)
  - Specific internal energy ($u = U/m$)
  - Specific enthalpy ($h = H/m$)
  - Specific entropy ($s = S/m$)

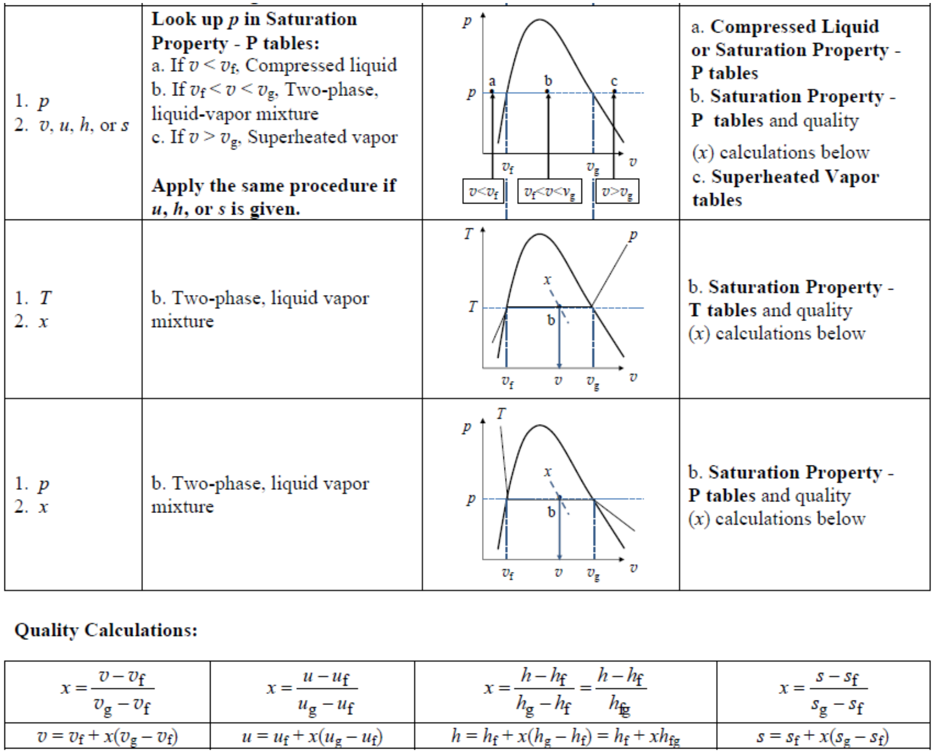
Enthalpy (h) is a property defined for convenience: **<u>used ONLY for mass flow in open systems!</u>** $h = u + Pv$

Property determination

1.) Think about what two, independent, intensive properties are available (state principle)
2.) Determine the phase of the substance (CL, Sat. L, SLVM, Sat. V, SHV)
3.) Select the appropriate table based on Step 2 – if CL data not available, use a suitable approximation



Given Properties	Test to Determine Region of Vapor Dome	Vapor Dome Diagram	Where/How to Obtain Properties
1. p 2. T	**Look up p in Saturation Properties - P tables:** a. If $T < T_{sat}$, Compressed liquid b. If $T = T_{sat}$, Two-phase, liquid-vapor mixture c. If $T > T_{sat}$, Superheated vapor d. If $T > T_c$, Superheated vapor		a. **Compressed Liquid** or **Saturation Property - T tables** b. T and p are not independent; need another property c.&d. **Superheated Vapor tables**
1. p 2. T	**Look up T in Saturation Properties - T tables:** a. If $p > p_{sat}$, Compressed liquid b. It $p = p_{sat}$, Two-phase, liquid-vapor mixture c. If $p < p_{sat}$, Superheated vapor		a. **Compressed Liquid or Saturation Property - T tables** b. T and p are not independent; need another property c. **Superheated Vapor tables**
1. T 2. v, u, h, or s	**Look up T in Saturation Property - T tables:** a. If $v < v_f$, Compressed liquid b. If $v_f < v < v_g$, Two-phase, liquid-vapor mixture c. If $v > v_g$, Superheated vapor **Apply the same procedure if u, h, or s is given.**		a. **Compressed Liquid or Saturation Property - T tables** b. **Saturation Property - T** and quality (x) calculations below c. **Superheated Vapor tables**



1. p 2. v, u, h, or s	**Look up p in Saturation Property - P tables:** a. If $v < v_f$, Compressed liquid b. If $v_f < v < v_g$, Two-phase, liquid-vapor mixture c. If $v > v_g$, Superheated vapor **Apply the same procedure if u, h, or s is given.**		a. **Compressed Liquid or Saturation Property - P tables** b. **Saturation Property - P tables** and quality (x) calculations below c. **Superheated Vapor tables**
1. T 2. x	b. Two-phase, liquid vapor mixture		b. **Saturation Property - T tables** and quality (x) calculations below
1. p 2. x	b. Two-phase, liquid vapor mixture		b. **Saturation Property - P tables** and quality (x) calculations below

Quality Calculations:

$x = \dfrac{v - v_f}{v_g - v_f}$	$x = \dfrac{u - u_f}{u_g - u_f}$	$x = \dfrac{h - h_f}{h_g - h_f} = \dfrac{h - h_f}{h_{fg}}$	$x = \dfrac{s - s_f}{s_g - s_f}$
$v = v_f + x(v_g - v_f)$	$u = u_f + x(u_g - u_f)$	$h = h_f + x(h_g - h_f) = h_f + xh_{fg}$	$s = s_f + x(s_g - s_f)$

Quality (summary) : an intensive thermodynamic property

- Quality (x) of the SLVM is defined as mass of vapor divided by the total mass of mixture

$$x = \frac{m_v}{m_{total}}$$

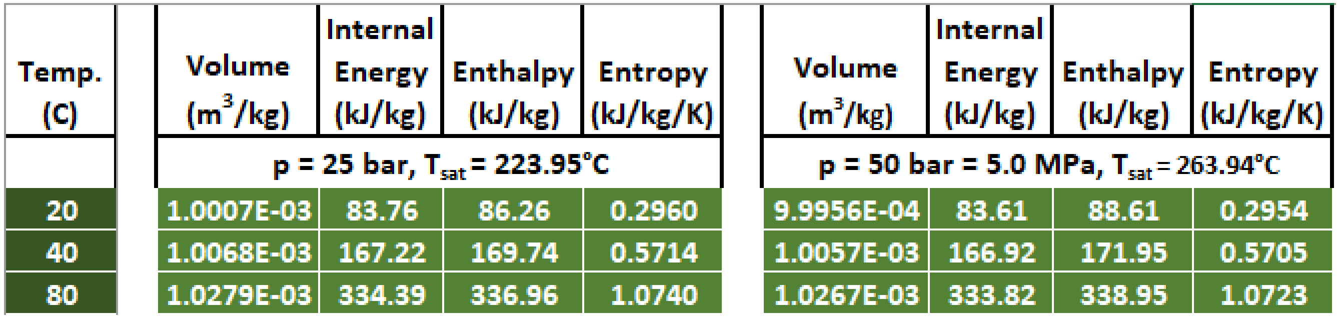
- x lies between 0 (saturated liquid) and 1 (saturated vapor)
- Quality is not relevant outside the vapor dome (i.e. no quality associated with either compressed liquid or superheated vapor states)

- At **<u>any</u>** point inside the vapor dome:
  - $v = v_f + x(v_g - v_f)$
  - $u = u_f + x(u_g - u_f)$
  - $h = h_f + x(h_g - h_f)$
  - $s = s_f + x(s_g - s_f)$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{u - u_f}{u - u_f} = \frac{h - h_f}{h_g - h_f} = \frac{s - s_f}{s_g - s_f}$$

CL tables

- Tables are organized by pressure and corresponding sat. temperature
- Properties are listed at various temperatures for a given pressure

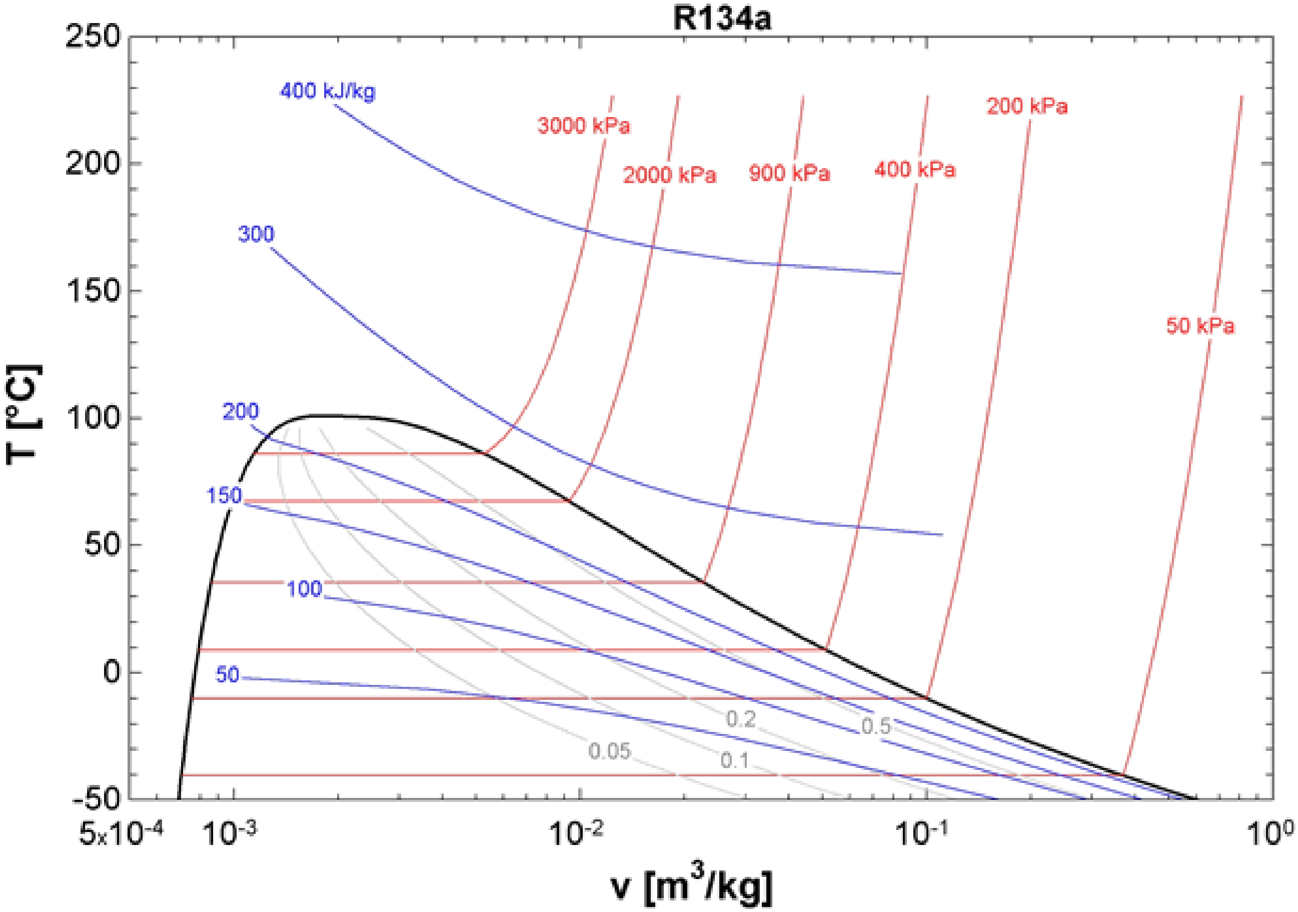


Temp. (C)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)	Volume (m^3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg/K)
	p = 25 bar, T_{sat} = 223.95°C				p = 50 bar = 5.0 MPa, T_{sat} = 263.94°C			
20	1.0007E-03	83.76	86.26	0.2960	9.9956E-04	83.61	88.61	0.2954
40	1.0068E-03	167.22	169.74	0.5714	1.0057E-03	166.92	171.95	0.5705
80	1.0279E-03	334.39	336.96	1.0740	1.0267E-03	333.82	338.95	1.0723

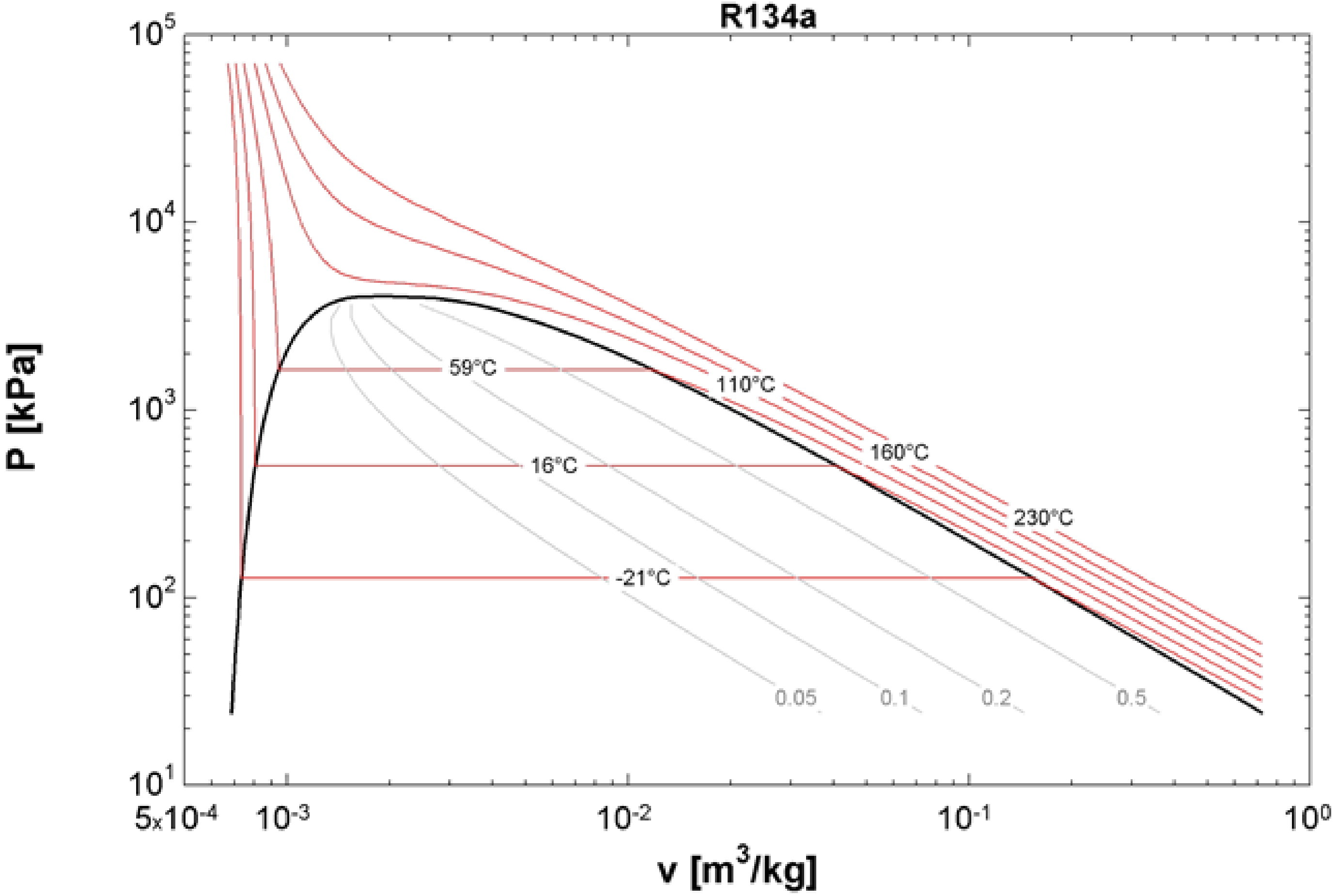
- Compressed liquid approximations (only use as last resort, i.e. no table exists)
  - Liquid can be considered incompressible if ΔP barely changes v and u
  - Under this **assumption**, properties at the saturated liquid state can be used as approximations:

$$u \approx u_f(T)$$
$$v \approx v_f(T)$$
$$h \approx u_f(T) + P v_f(T) \approx h_f(T) + v_f(T)[P - P_{sat}(T)]$$

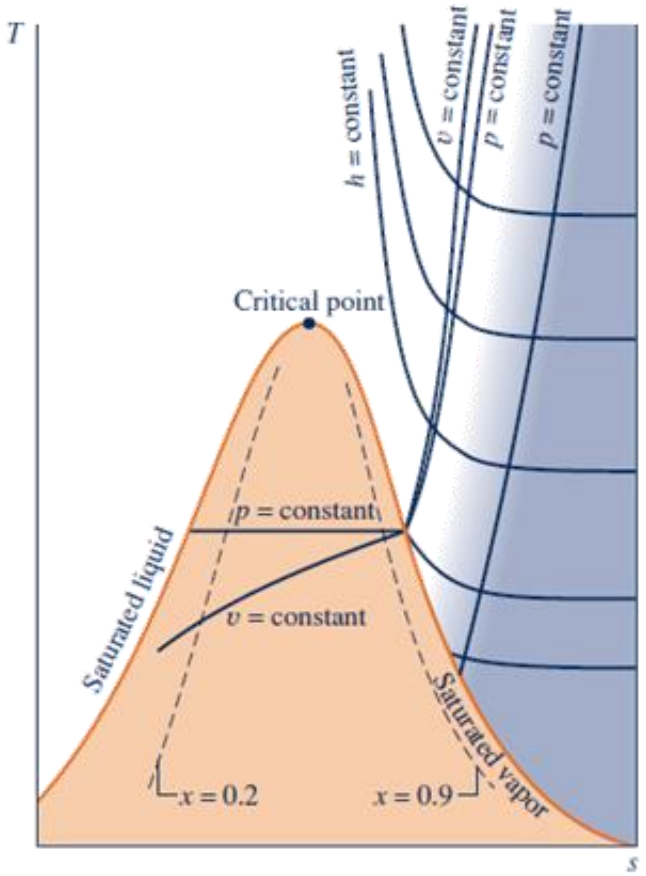
Thermodynamic Plots: Example T-v

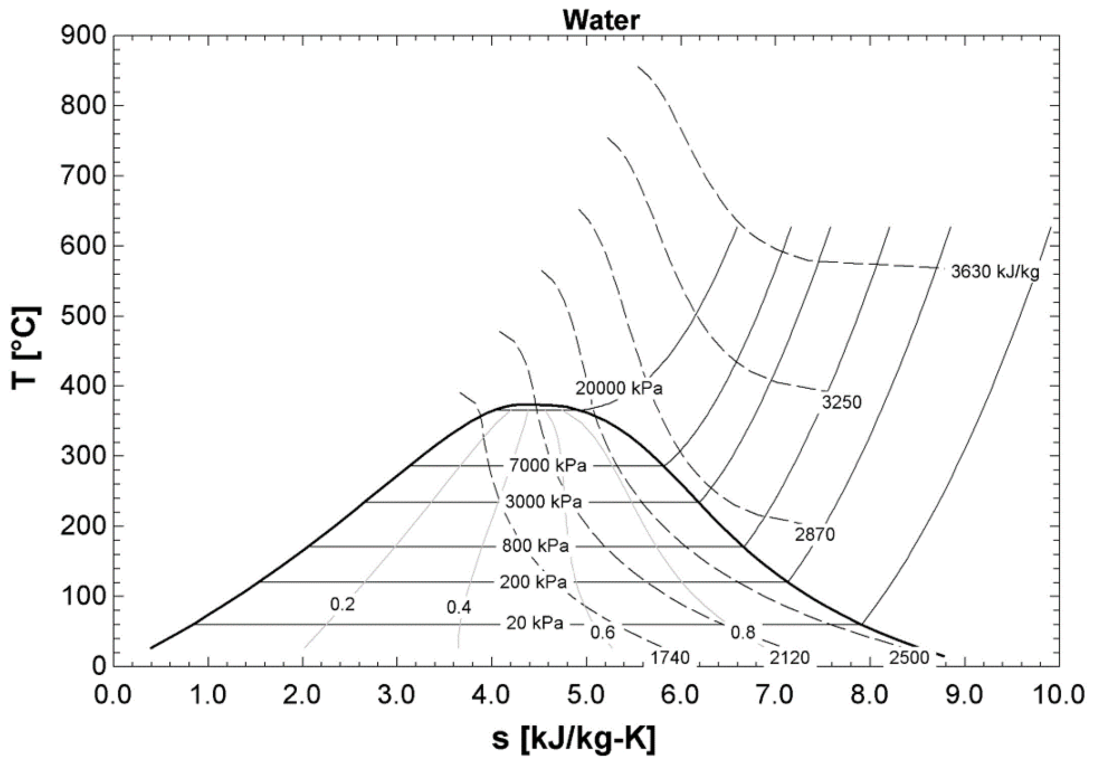


Thermodynamic Plots: Example P-v



Thermodynamic Plots: Example T-s





Specific heat

- Specific Heat: Heat (energy) required to rise the temperature of a unit mass of a substance by one degree.
- We will consider two types of specific heats:
  - Specific heat at constant volume (c_v)
  - Specific heat at constant pressure (c_p)

Specific heat at constant volume (c_v)

$$\boxed{c_v = \left(\frac{\partial u}{\partial T}\right)_{v=constant}}$$

In general, $u = f(P \text{ or } v, T)$

❖ c_v is related to change in internal energy per unit change in temperature at constant volume

Specific heat at constant pressure

$$\boxed{c_p = \left(\frac{\partial h}{\partial T}\right)_{p=constant}}$$

In general, $h = f(P \text{ or } v, T)$

❖ c_p is related to change in enthalpy per unit change in temperature at constant pressure

Summary: Incompressible substances

- Substances where volume changes are negligible
- Valid assumption for liquids and solids, **NOT GASES**
- $u = f(T)$ only since v is constant
- $c_p = c_v = c$
- $u_2 - u_1 = \int_{T_1}^{T_2} c(T)dT$
- $h_2 - h_1 = \int_{T_1}^{T_2} c(T)dT + v(P_2 - P_1)$

- Additionally, if we assume ***constant specific heats***:

$$u_2 - u_1 = c(T_2 - T_1)$$

$$h_2 - h_1 = c(T_2 - T_1) + v(P_2 - P_1)$$

Ideal Gas

- For a compressible substance, volume changes **are substantial** when pressure changes
- Ideal gas is an “imaginary” substance that has certain characteristics
  - Negligible particle volume
  - Negligible particle forces
- Is this possible? Not really! However, we can get very close to such behavior at very low density
- Usually applicable for <u>high temperature and low pressure</u>

Relative to critical point $P \ll P_{crit}$ and $T \gg T_{crit}$

Ideal gases – internal energy and enthalpy

- For ideal gases, internal energy depends *only* on temperature, $u = u(T)$:

$$c_v(T) = \frac{du}{dT}$$ ideal-gas specific heat at constant volume

$$\boxed{u_2(T_2) - u_1(T_1) = \int_{T_1}^{T_2} c_v(T)\,dT}$$ or $\underline{\Delta u = c_v(T_2 - T_1)}$ Only when c_v is constant from T_1 to T_2

- For ideal gases, enthalpy also depends only on temperature, $h = h(T)$:

$$h = u(T) + pv = u(T) + RT, \text{ R constant, so } h = h(T)$$

$$c_p(T) = \frac{dh}{dT}$$ ideal-gas specific heat at constant pressure

$$\boxed{h_2(T_2) - h_1(T_1) = \int_{T_1}^{T_2} c_p(T)\,dT}$$ or $\underline{\Delta h = c_p(T_2 - T_1)}$ Only when c_p is constant from T_1 to T_2

Evaluating internal energy and enthalpy changes for an ideal gas

- Use tabulated data: most accurate, we have ideal gas tables on BLACKBOARD

$$\Delta u = u_2(T_2) - u_1(T_1)$$
$$\Delta h = h_2(T_2) - h_1(T_1)$$

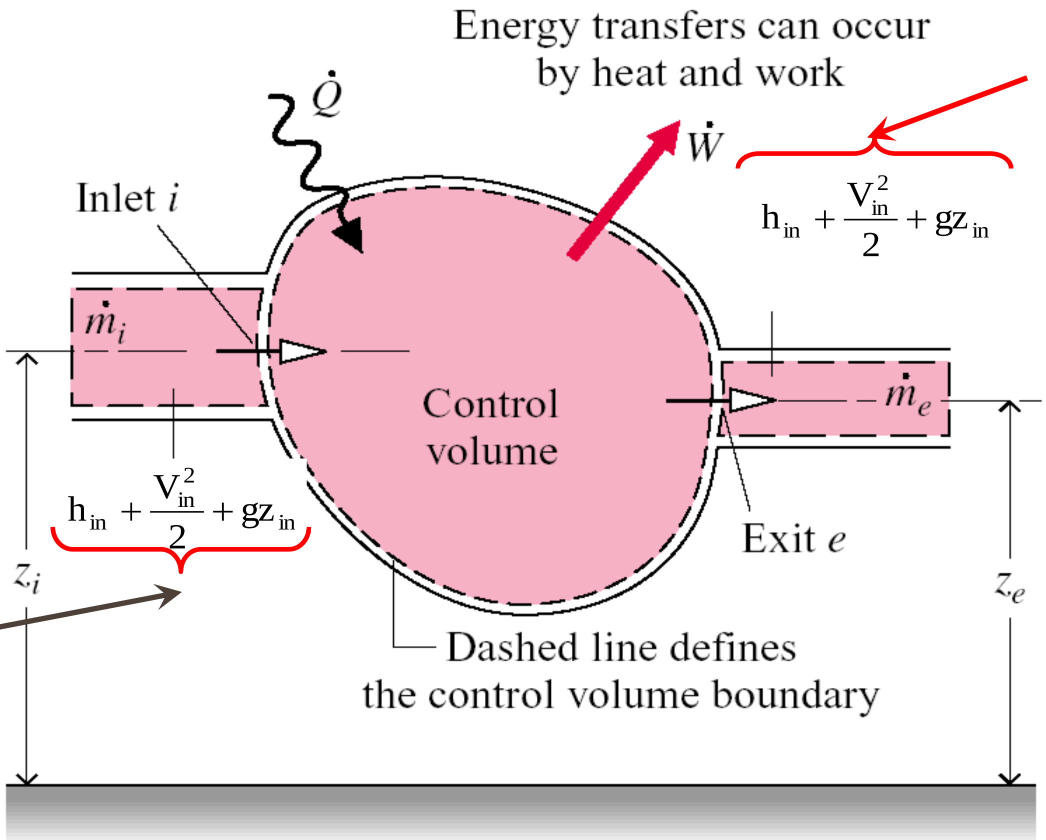
values at T_1 and T_2 from ideal gas tables

- Use constant specific heats: only works for very small temperature ranges, usually not a good assumption for ideal gases

$$\Delta u \approx c_v(T_2 - T_1)$$
$$\Delta h \approx c_p(T_2 - T_1)$$

Only when c_v and c_p are constant or almost constant for the temperature range

Control Volume: Open System



$$\underbrace{h_{in} + \frac{V_{in}^2}{2} + gz_{in}}$$

Energy exiting the control volume in the forms of kinetic energy, potential energy, and enthalpy (h = u + pv)

$$\underbrace{h_{in} + \frac{V_{in}^2}{2} + gz_{in}}$$

Energy entering the control volume in the forms of kinetic energy, potential energy, and enthalpy

$$\frac{dE_{C\forall}}{dt} = \dot{Q} - \dot{W} + \sum \dot{m}_{in}\left(h_{in} + \frac{V_{in}^2}{2} + gz_{in}\right) - \sum \dot{m}_{out}\left(h_{out} + \frac{V_{out}^2}{2} + gz_{out}\right)$$

At steady-state, $\frac{dE_{C\forall}}{dt} = 0$

Nozzles and Diffusers

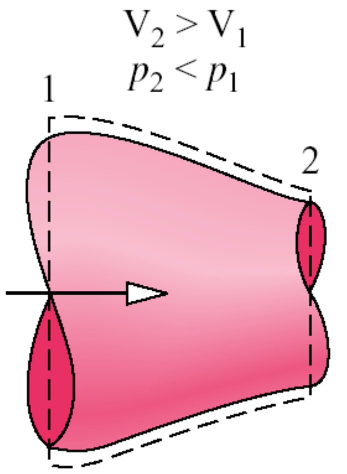
Assumptions:
(1) adiabatic, Q = 0
(2) no volume changes, W = 0
(3) steady-state, d/dt = 0
(4) change in potential energy negligible

$$\cancel{\frac{dE_{CV}}{dt}} = \cancel{\dot{Q}} - \cancel{\dot{W}} + \sum \dot{m}_{in}\left(h_{in} + \frac{V_{in}^2}{2} + \cancel{gz_{in}}\right) - \sum \dot{m}_{out}\left(h_{out} + \frac{V_{out}^2}{2} + \cancel{gz_{out}}\right)$$

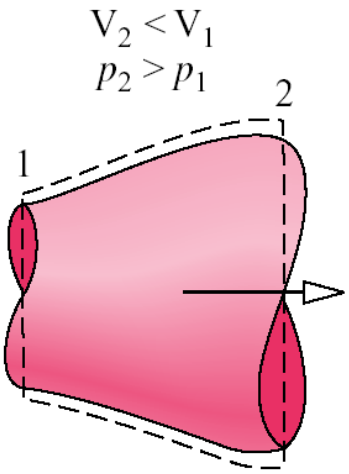
$$\boxed{h_{in} + \frac{V_{in}^2}{2} = h_{out} + \frac{V_{out}^2}{2}}$$

or

$$\boxed{\Delta h = h_{out} - h_{in} = \frac{V_{in}^2}{2} - \frac{V_{out}^2}{2}}$$



Nozzle

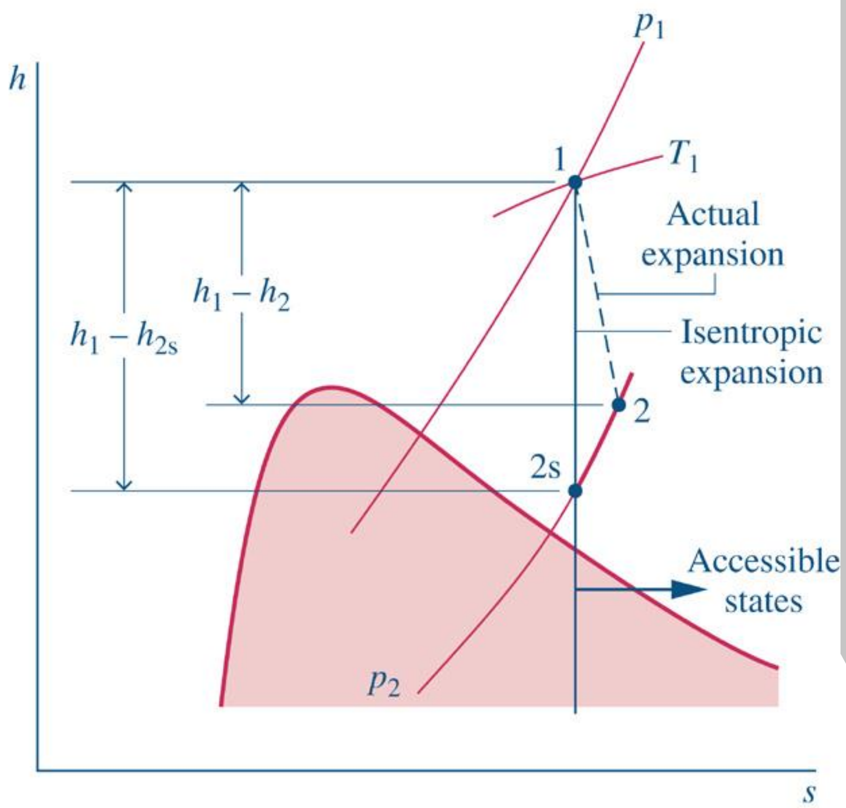


Diffuser

Nozzles and Diffusers

Nozzle Efficiency:

Compares the performance of a real nozzle or diffuser to the performance of an ideal, isentropic nozzle or diffuser operating *between the same pressures*



$$\eta_{nozzle} = \frac{\Delta h_{actual}}{\Delta h_{ideal}} = \frac{\left(V_{outlet}^2 - V_{inlet}^2\right)/2}{\left(V_{outlet,s}^2 - V_{inlet}^2\right)/2}$$

Actual process

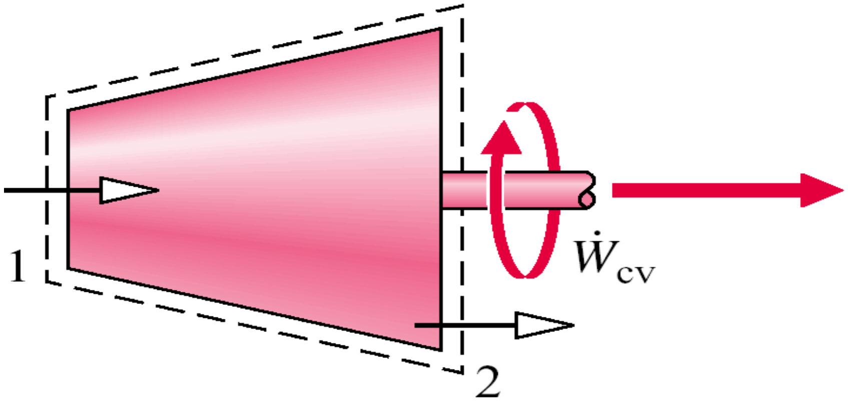
Isentropic process with same initial state and final pressure

Turbines, Pumps and Compressors

Assumptions: (1) adiabatic, $Q = 0$
(2) change in potential energy negligible
(3) steady-state, $d/dt = 0$
(4) change in kinetic energy negligible

$$\cancel{\frac{dE_{CV}}{dt}} = \cancel{\dot{Q}} - \dot{W} + \sum \dot{m}_{in}\left(h_{in} + \cancel{\frac{V_{in}^2}{2}} + \cancel{gz_{in}}\right) - \sum \dot{m}_{out}\left(h_{out} + \cancel{\frac{V_{out}^2}{2}} + \cancel{gz_{out}}\right)$$

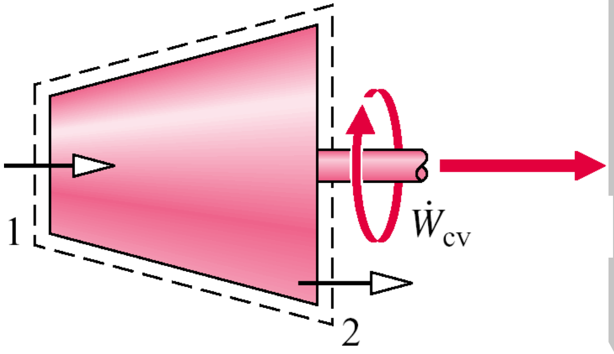
$$\boxed{\frac{\dot{W}}{\dot{m}} = h_{in} - h_{out}}$$



Turbines, Pumps and Compressors

Turbine and compressor/pump efficiencies both compare the performance of an actual device to the performance of an ideal, isentropic device operating *between the same pressures*

- Turbine Efficiency: In turbines, the actual power generation will be less than the ideal power generation



$$\eta_t = \frac{\Delta h_{actual}}{\Delta h_{ideal}} = \frac{\left(h_{inlet} - h_{outlet}\right)_{actual}}{\left(h_{inlet} - h_{outlet,s}\right)_s}$$

isentropic

- Pump/Compressor Efficiency: In pumps/compressors, the actual power consumption will be greater than the ideal power consumption

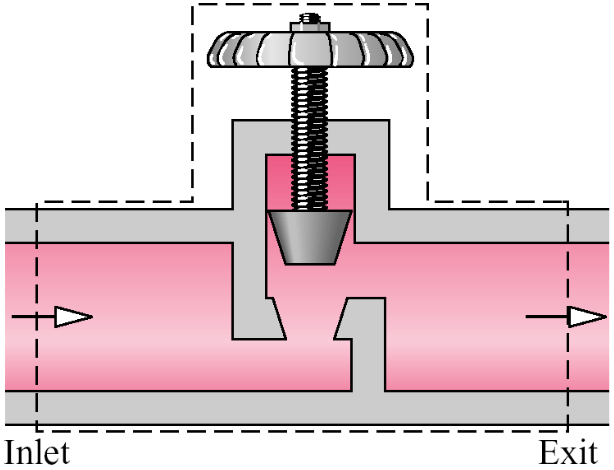
isentropic

$$\eta_c = \frac{\Delta h_{ideal}}{\Delta h_{actual}} = \frac{\left(h_{outlet} - h_{inlet,s}\right)_s}{\left(h_{outlet} - h_{inlet}\right)_{actual}}$$

Throttling Valves

Assumptions:
(1) adiabatic, Q = 0
(2) no volume changes, W = 0
(3) steady-state, d/dt = 0
(4) change in potential energy negligible
(5) change in kinetic energy negligible

$$\cancel{\frac{dE_{CV}}{dt}} = \cancel{\dot{Q}} - \cancel{\dot{W}} + \sum \dot{m}_{in}\left(h_{in} + \cancel{\frac{V_{in}^2}{2}} + \cancel{gz_{in}}\right) - \sum \dot{m}_{out}\left(h_{out} + \cancel{\frac{V_{out}^2}{2}} + \cancel{gz_{out}}\right)$$



$$\boxed{h_{in} = h_{out}}$$

Boilers, Condensers, and Evaporators

Assumptions:
(1) passive devices, W = 0
(2) change in potential energy negligible
(3) steady-state, d/dt = 0
(4) change in kinetic energy negligible

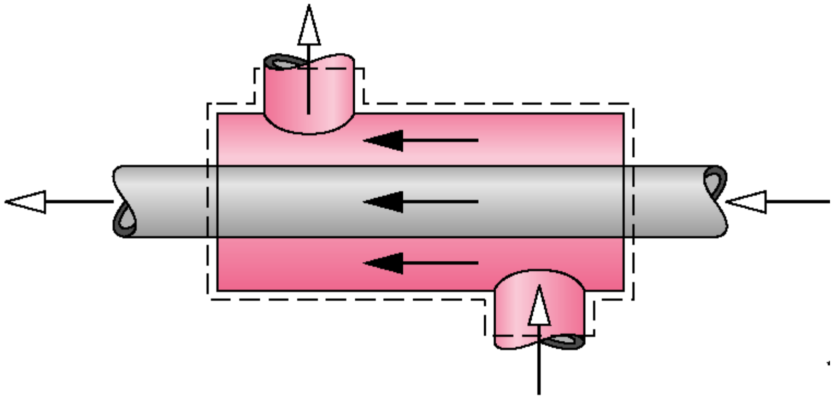
$$\cancel{\frac{dE_{CV}}{dt}} = \dot{Q} - \cancel{\dot{W}} + \sum \dot{m}_{in}\left(h_{in} + \cancel{\frac{V_{in}^2}{2}} + \cancel{gz_{in}}\right) - \sum \dot{m}_{out}\left(h_{out} + \cancel{\frac{V_{out}^2}{2}} + \cancel{gz_{out}}\right)$$

$$\boxed{\frac{\dot{Q}}{\dot{m}} = h_{out} - h_{in}}$$

Heat Exchangers

Assumptions:
(1) passive devices, W = 0
(2) change in potential energy negligible
(3) steady-state, d/dt = 0
(4) change in kinetic energy negligible
(5) adiabatic, Q = 0

$$\cancel{\frac{dE_{CV}}{dt}} = \cancel{\dot{Q}} - \cancel{\dot{W}} + \sum \dot{m}_{in}\left(h_{in} + \cancel{\frac{V_{in}^2}{2}} + \cancel{gz_{in}}\right) - \sum \dot{m}_{out}\left(h_{out} + \cancel{\frac{V_{out}^2}{2}} + \cancel{gz_{out}}\right)$$



$$\boxed{\sum \dot{m}_{in} h_{in} = \sum \dot{m}_{out} h_{out}}$$

Reversible processes and irreversible processes

- **Reversible Process** is a process in which both system as well as all parts of the surroundings can be exactly restored to their respective initial states after the process has taken place

- **Irreversible Process** is a process in which both system as well as all parts of the surroundings cannot be exactly restored to their respective initial states after the process has taken place

All actual processes are irreversible; we try to minimize factors causing irreversibility to optimize performance

Second law Statements

- Clausius Statement
  - *It is impossible to construct a device that operates in a cycle and produces no effects other than the transfer of heat from a lower temperature body to a higher temperature body*
- Kelvin-Planck Statement
  - *It is impossible for any system to operate in a thermodynamic cycle and deliver a net amount of energy by work to its surroundings while receiving energy by heat transfer from a single reservoir*
- **Entropy Statement**
  - ***It is impossible for any system to operate in a way that entropy is destroyed***

Entropy Statement is our main focus in ME200

Second law corollaries

The thermal efficiency/COP of an irreversible cycle is always less than the thermal efficiency/COP of a reversible cycle when each are operated between the same two thermal reservoirs.

$$\eta_{th}\begin{cases} < \eta_{th,rev} & \text{irreversible cycle} \\ = \eta_{th,rev} & \text{reversible cycle} \\ > \eta_{th,rev} & \text{impossible cycle} \end{cases} \qquad COP\begin{cases} < COP_{rev} & \text{irreversible cycle} \\ = COP_{rev} & \text{reversible cycle} \\ > COP_{rev} & \text{impossible cycle} \end{cases}$$

All **reversible** cycles operating between the same two thermal reservoirs have the same thermal efficiency/COP.

Implies that η_{th} a function of T_C and T_H only for a reversible cycles

$$\eta_{th} = 1 - \left(\frac{Q_C}{Q_H}\right)_{rev,1} = 1 - \left(\frac{Q_C}{Q_H}\right)_{rev,2}$$

Second law corollaries

- Power cycle

$$\boxed{\eta_{rev} = 1 - \frac{T_C}{T_H}}$$

Any real power cycle will have a thermal efficiency less than this maximum reversible thermal efficiency

- Heat pump cycle

$$\boxed{COP_{HP,rev} = \frac{T_H}{T_H - T_C}}$$

Any real heat pump cycle will have a COP less than this maximum reversible COP

- Refrigeration cycle

$$\boxed{COP_{R,rev} = \frac{T_C}{T_H - T_C}}$$

Any real refrigeration cycle will have a COP less than this maximum reversible COP

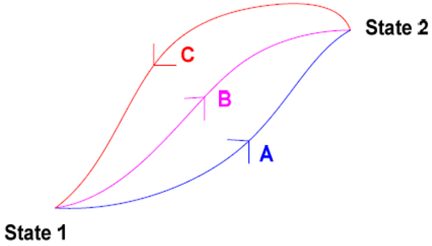
Clausius inequality

$$\oint\left(\frac{\delta Q}{T}\right)_b = -\sigma_{cycle} \leftarrow$$

defined as entropy generation

We define this property to be **entropy!**

$\sigma_{cycle} = 0$, reversible cycle
$\sigma_{cycle} > 0$, irreversible cycle
$\sigma_{cycle} < 0$, **impossible** cycle



- Consider two cycles between state 1 and state 2
- Cycle I: Process A and C (both internally reversible)
- Cycle II: Process B and C (both internally reversible)

Does the cyclic integral $\oint\left(\frac{\delta Q}{T}\right)_b$ depend on the specific path?

No – it doesn't depend on path. Therefore, the integral must represent a change in some property!

Entropy

- Based on the above, we define entropy (S)

$$dS = S_2 - S_1 = \left(\int_1^2 \frac{\delta Q}{T}\right)_{int.rev.}$$

- Entropy is an extensive **<u>property</u>** – units J/K
- Specific entropy (s) – entropy per unit mass, units J/(kg-K)

- For most real substances, entropy in “looked up” in tables just like our other properties!

$T - ds$ relations

- Evaluating $dS = \left(\frac{\delta Q}{T}\right)_{\text{int. rev}}$ not always practical
- $T - dS$ relations are a way to obtain specific entropy from other properties
- $T - dS$ relations are:

$$\boxed{\begin{aligned} Tds &= du + Pdv \\ Tds &= dh - vdP \end{aligned}}$$

Ideal gas entropy lookups

- For ideal gases, entropy is a function of both *T* and *P*

$$s_2 - s_1 = \int_{T_1}^{T_2} c_P(T)\frac{dT}{T} - R\ln\left(\frac{P_2}{P_1}\right)$$

$c_p(T)$ integral is hard to evaluate

Define $s^\circ(T) = \int_{T'}^{T} c_P(T)\frac{dT}{T}$, then $\int_{T_1}^{T_2} c_P(T)\frac{dT}{T} = s^\circ(T_2) - s^\circ(T_1)$

Therefore $$\boxed{s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - R\ln\left(\frac{P_2}{P_1}\right)}$$

$s^\circ(T)$ looked up ideal gas tables

Evaluating changes in entropy

- "Real" substances (i.e. water) – look up in tables just like other properties such as u, h, v, etc.

- For incompressible substance: $dS = \frac{c(T)}{T} dT$, or $s_2 - s_1 = c \ln\left(\frac{T_2}{T_1}\right)$

- For ideal gases, we use the relationship $s_2 - s_1 = s^{\circ}(T_2) - s^{\circ}(T_1) - R \ln\left(\frac{P_2}{P_1}\right)$

For ideal gases, entropy is a function of both temperature and pressure – the values for $s^{\circ}(T)$ are tabulated in ideal gas tables

General entropy balance

Unlike mass and energy, entropy is not conserved!

Rate of entropy transfer accompanying heat transfer across the system boundary at temperature T_i

Rate of entropy transfer associated with mass leaving the system boundary

$$\underbrace{\frac{dS_{CV}}{dt}} = \overbrace{\sum_i \frac{\dot{Q}_i}{T_i}} + \underbrace{\sum_{in} \dot{m}_{in} s_{in}} - \overbrace{\sum_{out} \dot{m}_{out} s_{out}} + \underbrace{\dot{\sigma}}$$

Rate of change of entropy within the system boundary

Rate of entropy transfer associated with mass entering the system boundary

Rate of entropy production (generation) within the system

Entropy balance: $$\frac{dS}{dt} = \sum_i \frac{\dot{Q}_i}{T_i} + \sum_{in} \dot{m}_{in} s_{in} - \sum_{out} \dot{m}_{out} s_{out} + \dot{\sigma}$$

Control volume at steady-state
(single inlet, single outlet)

$$\cancel{\frac{dS}{dt}} = \sum_i \frac{\dot{Q}_i}{T_i} + \dot{m}_{in} s_{in} - \dot{m}_{out} s_{out} + \dot{\sigma}$$

$\dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}$

$$0 = \sum_i \frac{\dot{Q}_i}{T_i} + \dot{m}_{in} s_{in} - \dot{m}_{out} s_{out} + \dot{\sigma}$$

$$0 = \frac{\dot{Q}_i}{T_i} + \dot{m}(s_{in} - s_{out}) + \dot{\sigma}$$

Assuming only one heat transfer term

Closed system (control mass)

no $\dot{m}$

$$\frac{dS}{dt} = \sum_i \frac{\dot{Q}_i}{T_i} + \cancel{\dot{m}_{in} s_{in}} - \cancel{\dot{m}_{out} s_{out}} + \dot{\sigma}$$

$$\int_1^2 \frac{dS}{dt} = \int_1^2 \frac{\dot{Q}_i}{T_i} + \int_1^2 \dot{\sigma}$$

integrate

$$S_2 - S_1 = \frac{Q_{12}}{T_b} + \sigma_{12}$$

Assuming constant boundary temperature T_b

Adiabatic: no heat transfer → $Q, \dot{Q} = 0$

Internally Reversible: no entropy generation → $\sigma, \dot{\sigma} = 0$

Isentropic: closed system → $s_2 = s_1$
open system → $s_{in} = s_{out}$

2nd Law: requires $\sigma \geq 0$ and $\dot{\sigma} \geq 0$

Transient process:

$$\frac{dS}{dt} = \sum_i \frac{\dot{Q}_i}{T_i} + \sum_{in} \dot{m}_{in} s_{in} - \sum_{out} \dot{m}_{out} s_{out} + \dot{\sigma}$$

integrate

$$\int_1^2 \frac{dS}{dt} = \int_1^2 \frac{\dot{Q}_i}{T_i} + \int_1^2 \dot{m}_{in} s_{in} - \int_1^2 \dot{m}_{out} s_{out} + \int_1^2 \dot{\sigma}$$

$$m_2 s_2 - m_1 s_1 = \frac{Q_{12}}{T_b} + m_{in} s_{in} - m_{out} s_{out} + \sigma_{12}$$

Isentropic processes

- For a pure substance, use $\Delta s = 0$ to relate isentropic exit condition to inlet condition
- For an ideal gas

$$s_{2s} - s_1 = s^{\circ}_{2s} - s^{\circ}_1 - R\ln\left(\frac{P_{2s}}{P_1}\right) = 0$$

$$\frac{P_2}{P_1} = \frac{P_{r2s}}{P_{r1}} \qquad \frac{v_2}{v_1} = \frac{v_{r2s}}{v_{r1}}$$

Relative pressure

Relative volume

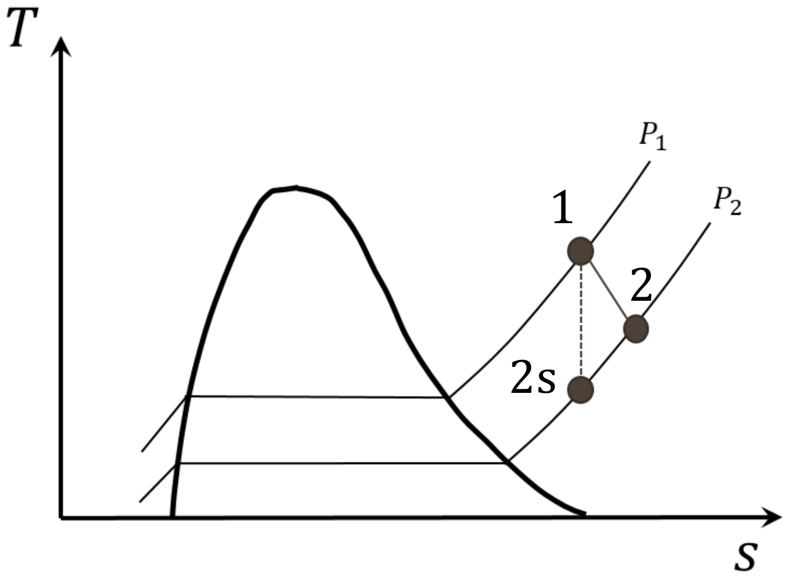
Note: All three relationships are equivalent to each other

Isentropic device efficiencies

State "2s" has entropy $s_{2s} = s_1$ and pressure $P_{2s} = P_2$

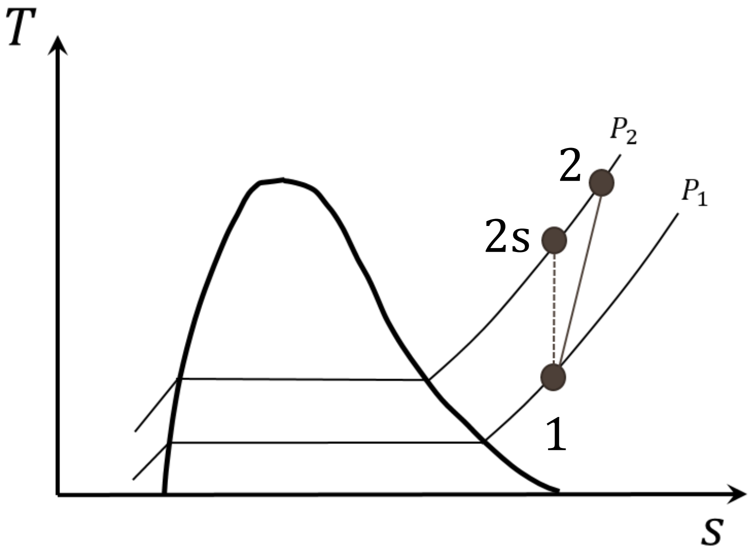
Turbine

$$\eta_t = \frac{(w_t)_{actual}}{(w_t)_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

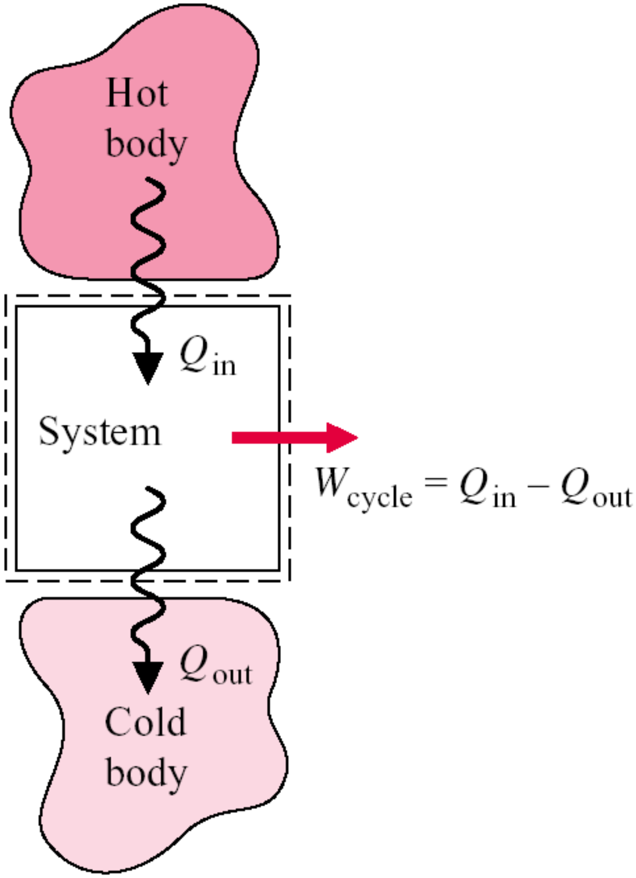


Compressor/pump

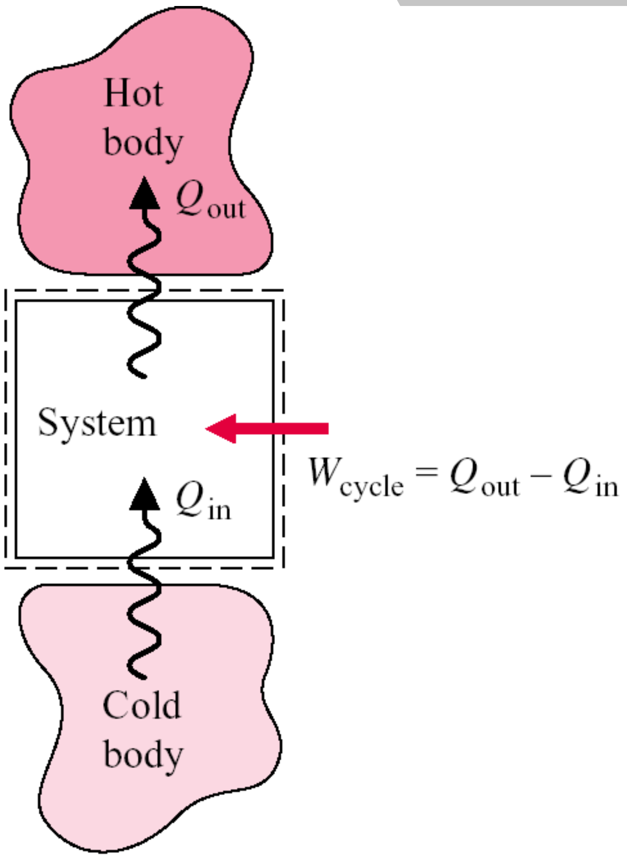
$$\eta_c = \frac{(w_c)_s}{(w_c)_{actual}} = \frac{h_{2s} - h_1}{h_2 - h_1}$$



Basic Cycles



Power Generation (Heat Engine)



Refrigeration, Air-Conditioning, Heat Pump

Basic Cycles: Definitions

Cycle: Series of processes that eventually brings the system back to its original state

Power Cycle:

- A cycle that uses heat energy to do work on the surroundings
- Performance is measured by *thermal efficiency*, which is the ratio of useful work output to the energy input required to run the cycle:

work done BY system

work done ON system

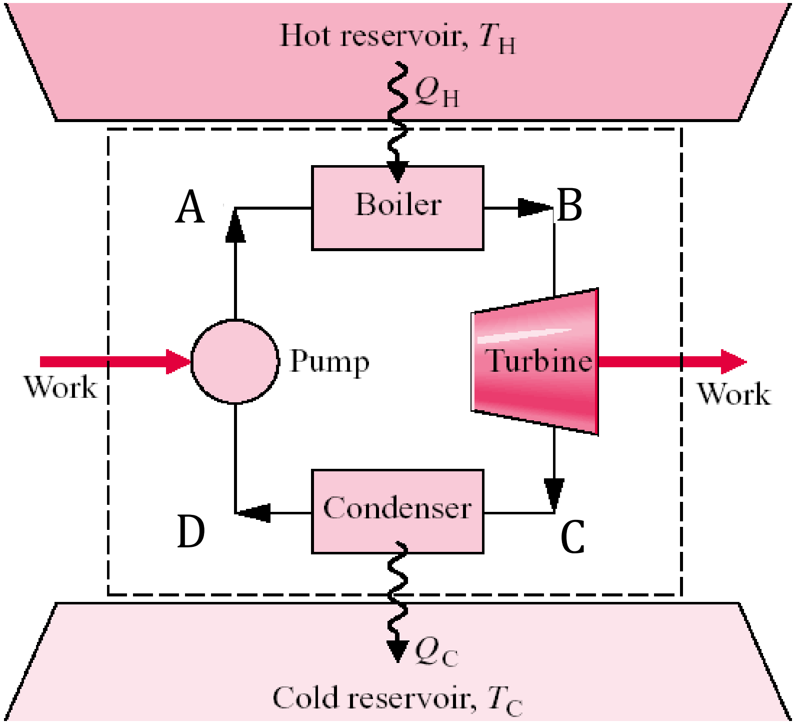
$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{W_{out} - W_{in}}{Q_{in}}$$

heat OUT OF system

$$= \frac{Q_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

heat INTO system

Carnot Cycles



Carnot Cycle:

(1) Theoretical implementation of a cycle without any irreversibilities

(2) Work output is the maximum possible for any heat engine due to reversible processes

Processes

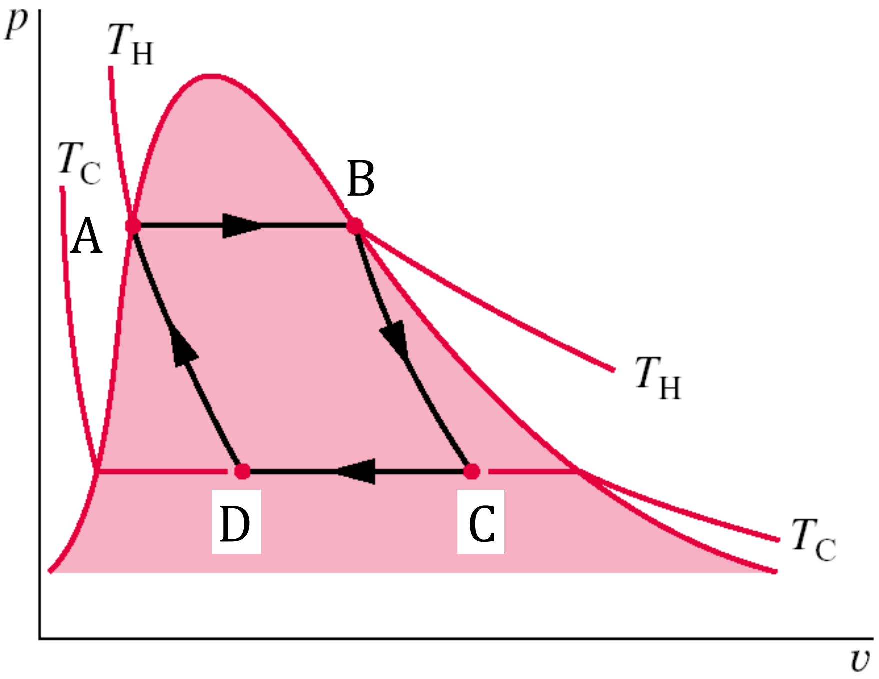
A to B: isothermal evaporation of saturated liquid to saturated vapor

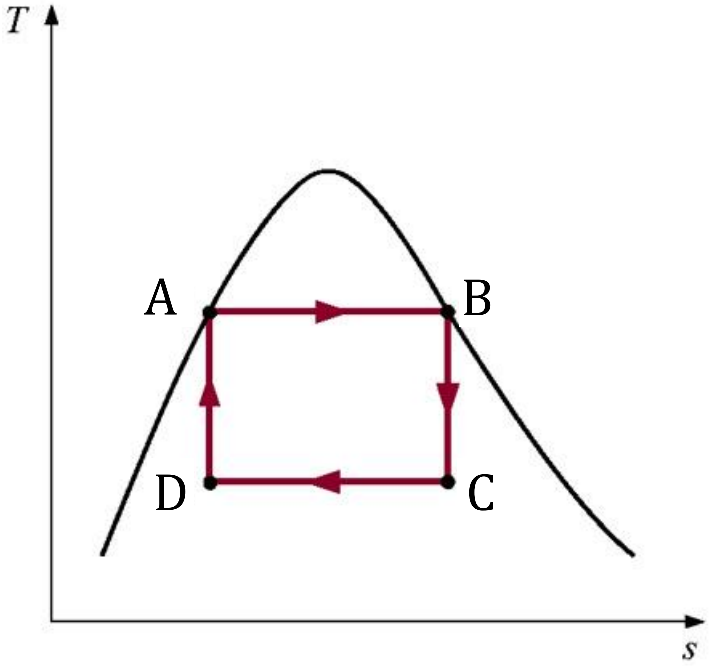
B to C: isentropic expansion of vapor ($Q = 0$; $\Delta s = 0$)

C to D: isothermal condensation of vapor

D to A: isentropic compression of vapor ($Q = 0$; $\Delta s = 0$)

Carnot P-v and T-s Diagrams





Carnot Efficiency:

$$\eta_{\text{th,Carnot}} = \frac{T_H - T_L}{T_H} = 1 - \frac{T_L}{T_H}$$

cold reservoir temperature

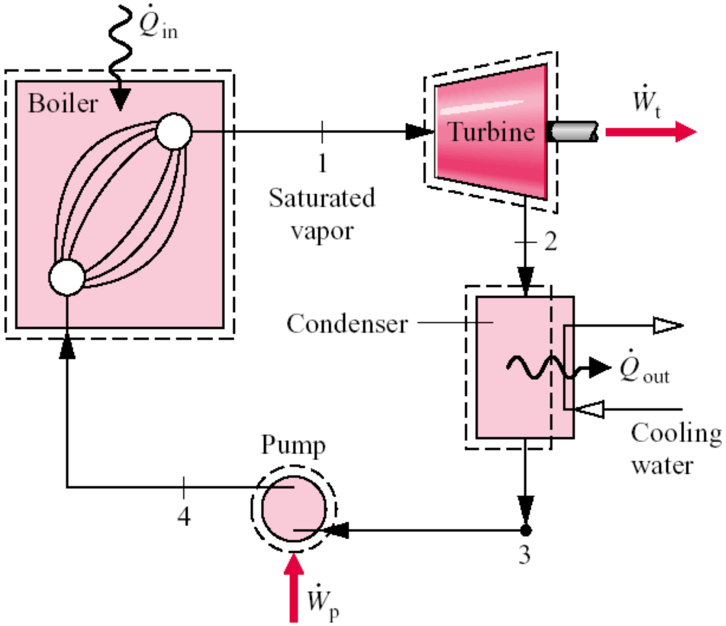
hot reservoir temperature

Rankine Cycle

Rankine Cycle:

- Vapor-power cycle commonly used in power plants with water as the working fluid
- Efficiency is ratio of useful output to required input:

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$



Processes

1 to 2: Isentropic expansion of the working fluid through the turbine from saturated vapor at state 1 to the condenser pressure (Q = 0; Δs = 0)

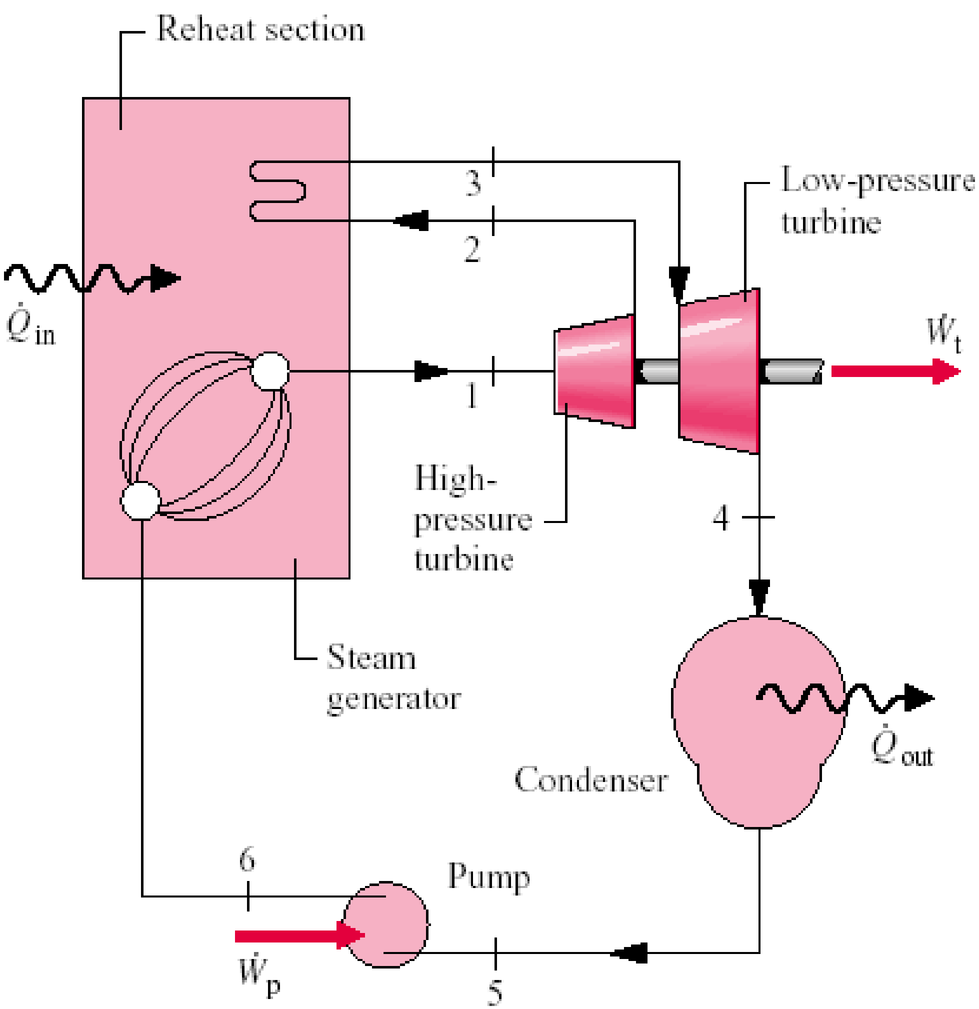
2 to 3: Heat transfer *from* the working fluid as it flows at constant pressure through the condenser with saturated liquid at state 3

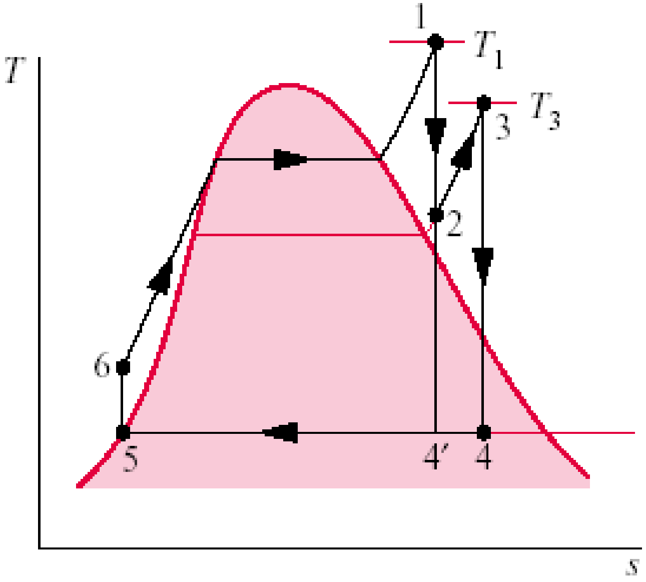
3 to 4: Isentropic compression in the pump to state 4 in the compressed liquid region. (Q = 0; Δs = 0)

4 to 1: Heat transfer *to* the working fluid as it flows at constant pressure through the boiler to complete the cycle

Rankine Superheat

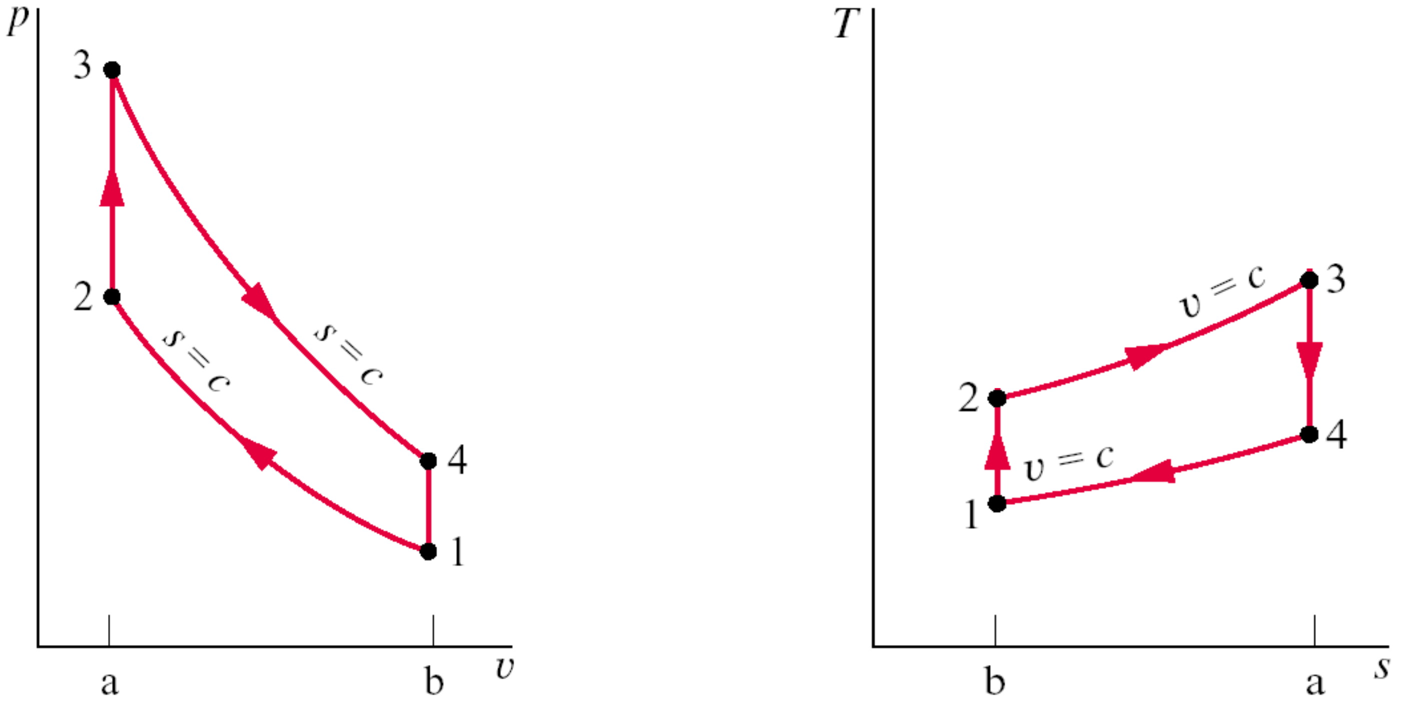
Superheat: Used to increase the temperature of the working fluid entering the low-pressure turbine to reduce the wear on turbine blades (point 3)





Air Standard Otto Cycles

Air-Standard Otto Cycle: Hypothetical closed system using air as the working fluid to simplify the chemistry due to combustion



Processes

1 to 2: Isentropic compression of the working fluid ($Q = 0$; $\Delta s = 0$)
2 to 3: Constant volume heat addition
3 to 4: Isentropic expansion of the working fluid ($Q = 0$; $\Delta s = 0$)
4 to 1: Constant volume heat rejection

Air Standard Otto Cycle: Performance

Compression Ratio:

$$r = \frac{\forall_1}{\forall_2} = \frac{\forall_4}{\forall_3} = \frac{\forall_b}{\forall_a}$$

compression from A to B

expansion from C to D

Cold-air analysis thermal efficiency:

specific heat ratio

$$\eta_{th} = 1 - r^{1-k}$$

compression ratio

Diesel Cycle

- Air-standard Diesel cycle for CI engines:
- Constant pressure heat addition
  - Compression Ratio:

$$r = \frac{V_1}{V_2} = \frac{V_{BDC}}{V_{TDC}}$$

  - Cutoff Ratio:

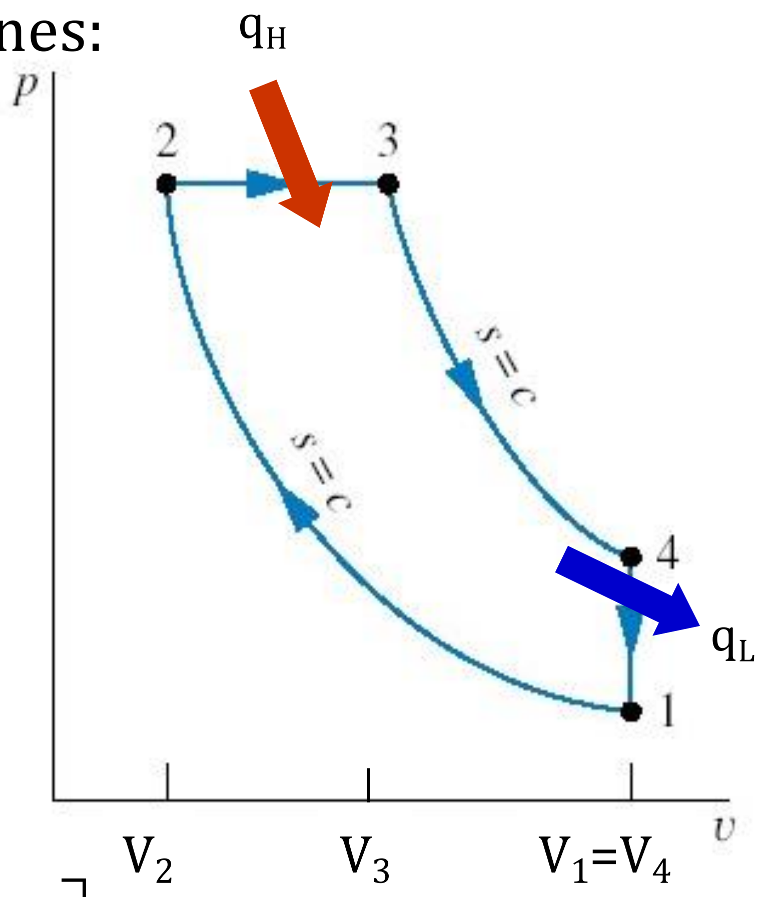
$$r_c = \frac{V_3}{V_2}$$

  - Thermal efficiency:

$$\eta_{th} = 1 - \frac{u_4 - u_1}{h_3 - h_2}$$

  - Assuming COLD air-standard

$$\eta_{th} = 1 - \frac{1}{r^{k\text{-}1}}\left[\frac{r_c^{k} - 1}{k\left(r_c - 1\right)}\right]$$



Ideal Brayton Cycle

- Isentropic compression and expansion
- Constant pressure heat addition and rejection
  - Back work ratio (compressor to turbine work ratio)

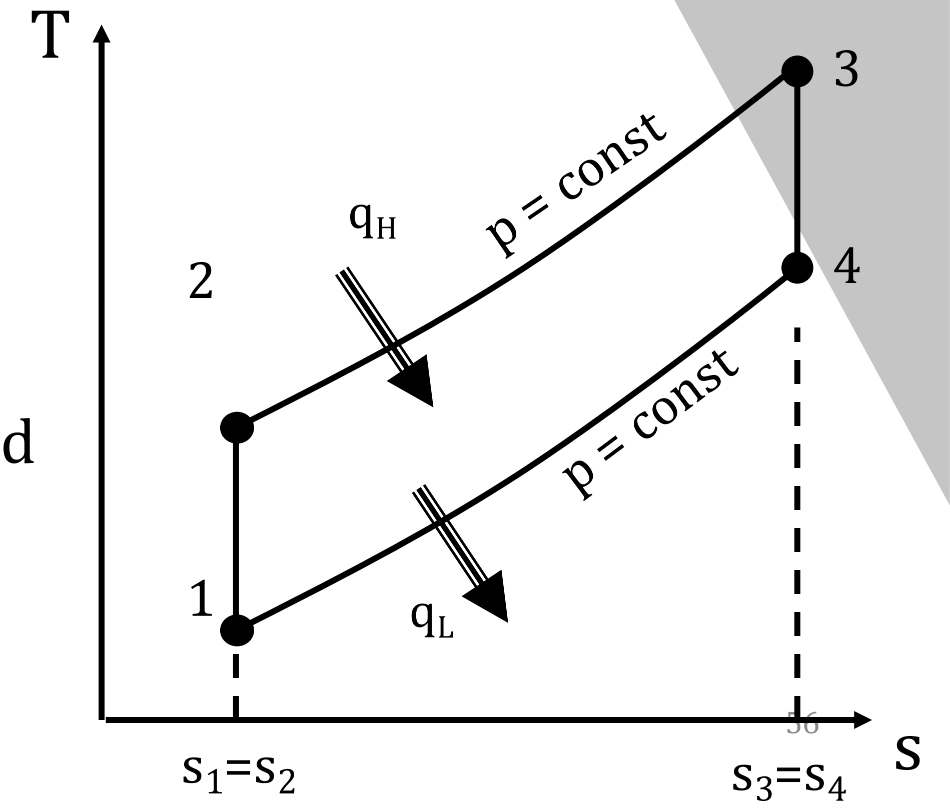
$$bwr = \frac{w_c}{w_t} = \frac{h_2 - h_1}{h_3 - h_4}$$

  - Thermal efficiency

$$\eta_{th} = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

  - Assuming COLD air-standard

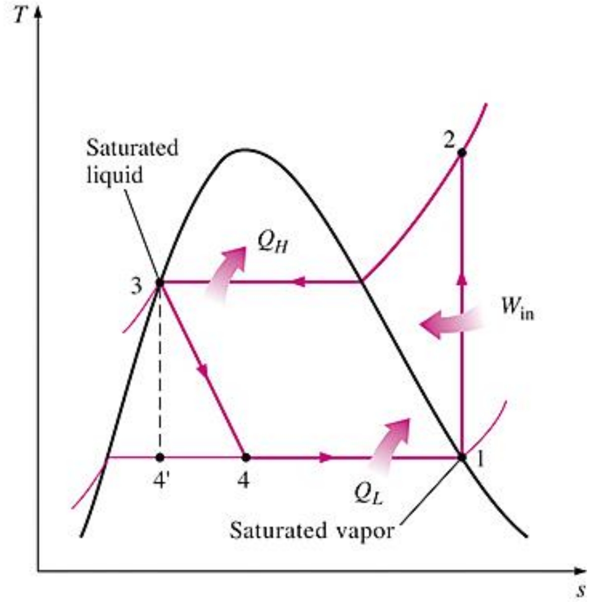
$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\left(p_2 / p_1\right)^{(k-1)/k}}$$

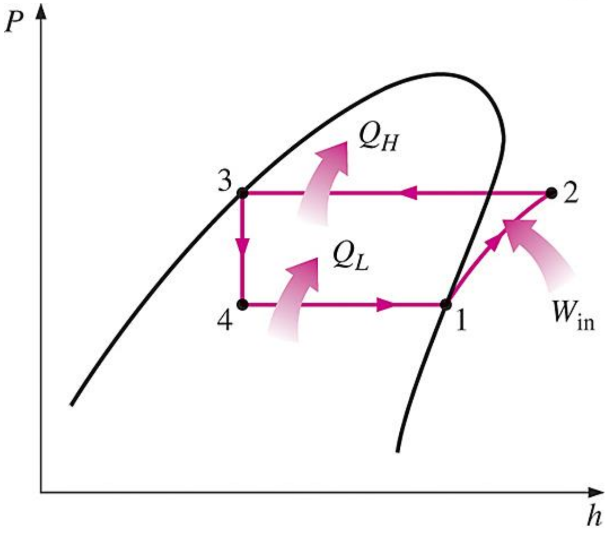


Refrigeration Cycles

- In refrigeration cycles, heat is transferred from a low-temperature area (i.e. inside the refrigerator) to a high-temperature area (e.g., in the kitchen)
  - Since heat spontaneously flows only from high to low temperature areas, work is required to force heat transfer
  - Opposite of heat engines
- Systems:
  - Refrigerator: Heat is removed from air inside
  - Air conditioner: Heat is removed from air in an occupied space
  - Heat Pump: Heat is supplied to air in an occupied space
  - Chiller: Heat is removed from water

Ideal Vapor-Compression Cycle





- Isentropic compression in a compressor (1-2)
- Isobaric heat rejection in a condenser (2-3)
- Adiabatic expansion in a throttling device (3-4)
- Isobaric heat addition in the evaporator (4-1)

Compressor

$$w_{1-2} = h_1 - h_2$$

$$h_1 = h\left(p_1, x_1 = 1\right)$$

$$s_2 = s_1$$

$$h_2 = h\left(p_2, s_2\right)$$

Condenser

$$q_{2-3} = q_H = h_3 - h_2$$

$$h_2 = h\left(p_2, s_2\right)$$

$$h_3 = h\left(p_3, x = 0\right)$$

$$p_3 = p_2$$

Expansion Valve

$$h_4 = h_3$$

$$p_3 = p_2$$

$$p_4 = p_1$$

$$x_4 = x\left(p_4, h_4\right)$$

Evaporator

$$q_{4-1} = q_L = h_1 - h_4$$

Refrigeration Cycles: Performance

- In refrigeration cycles, the *coefficient of performance* (COP) is used in place of thermal efficiency to measure performance
- The COP is always the ratio of useful energy transfer to the work input

$$COP_{\text{refrigerator,AC}} = \frac{Q_L}{W}$$

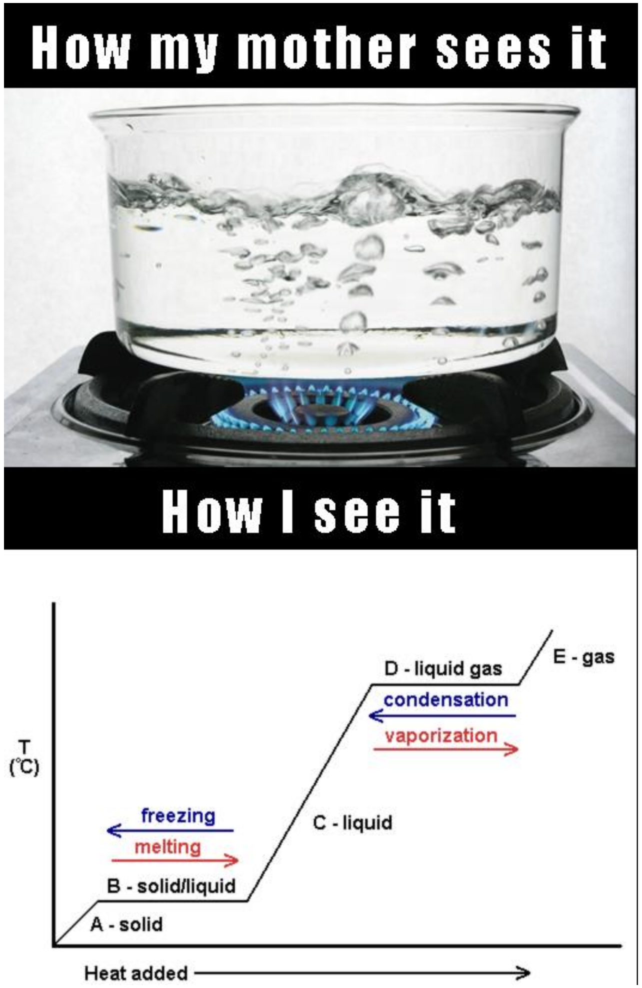
heat transfer from low-temperature reservoir

work input

$$COP_{\text{heat pump}} = \frac{Q_H}{W}$$

heat transfer from high-temperature reservoir

The End


How my mother sees it
How I see it
T
(°C)
A - solid
B - solid/liquid
freezing
melting
C - liquid
D - liquid gas
condensation
vaporization
E - gas
Heat added