Sections 29.1 to 29.7, 30.1 to 30.3

Experimentally, we find $\boxed{\mathcal{E} = -N\frac{d\Phi_B}{dt}}$, called Faraday's law of induction.

$\mathcal{E}$ is the **induced emf** (in V)—a potential difference that *may* give an **induced current**.

N *is* the number of turns (no unit).

Φ_B is the average magnetic flux (in Wb) through each turn at some time t (in s).

$\frac{d\Phi_B}{dt}$ is the *rate* at which Φ_B is changing with time (in $\frac{\text{Wb}}{\text{s}}$). Thus $1\ \frac{\text{Wb}}{\text{s}} = 1$ V.

Lenz's law: The direction of any magnetic induction effect is such as to oppose the cause of the effect.

The derivation on page 969 gives a relation that we can write as $\boxed{\mathcal{E} = v_\perp B_\perp l_\perp}$.

$\mathcal{E}$ is the (motional) emf (in V).

$v_\perp$, $B_\perp$, and $l_\perp$ are the *mutually perpendicular components* of the velocity $\vec{v}$ (in $\frac{\text{m}}{\text{s}}$), the uniform magnetic field $\vec{B}$ (in T), and the length vector $\vec{l}$ (in m) of a straight segment.

Cover up the solutions and carefully work Examples 29.1 to 29.5 and 29.7 to 29.10.

Since the emf is a potential difference, and a potential difference equals the integral of $\vec{E}\cdot d\vec{l}$, for a single loop Faraday's law of induction becomes $\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$.

This $\vec{E}$ is the *induced electric field* (in $\frac{\text{V}}{\text{m}}$ or $\frac{\text{N}}{\text{C}}$).

Cover up the solution and carefully work Example 29.11.

Eddy currents are induced currents that circulate within the volume of a conducting material.

Trying to apply Ampere's law ($\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$) in Fig. 29.21 gives $I_{\text{encl}} = i_{\text{C}}$ through the plane surface, but $I_{\text{encl}} = 0$ through the bulging surface. Maxwell resolved this contradiction by defining a quantity in the dielectric called the **displacement current** i_{D}: $\boxed{i_{\text{D}} \equiv \varepsilon\frac{d\Phi_E}{dt}}$. The displacement current is *not* an actual motion of charge, but has the SI unit of ampere = amp = A. Note $i_{\text{D}} = 0$ *only* if the electric flux Φ_E is constant.

ε is the permittivity (in $\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$ or $\frac{\text{F}}{\text{m}}$).

Φ_E is the electric flux (in $\frac{\text{N}\cdot\text{m}^2}{\text{C}}$ or V·m).

$\frac{d\Phi_E}{dt}$ is the *rate* at which the electric flux changes with time (in $\frac{\text{N}\cdot\text{m}^2}{\text{C}\cdot\text{s}}$ or $\frac{\text{V}\cdot\text{m}}{\text{s}}$).

Thus, when no magnetic materials are present, Ampere's law becomes $\oint \vec{B}\cdot d\vec{l} = \mu_0(i_{\text{C}} + i_{\text{D}})_{\text{encl}}$.

i_{C} is the enclosed conduction current (in A) (caused by actual motion of charge)

i_{D} is the enclosed displacement current (in A) (caused by changing enclosed electric flux).

The four **Maxwell's equations** summarize classical electromagnetism. The first four questions on your *final* exam will involve matching verbal descriptions to these four equations. For no dielectric or magnetic materials, Maxwell's equations reduce to:

$\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$, telling us electric field lines can start on positive charges and end on negative charges.

$\oint \vec{B} \cdot d\vec{A} = 0$, telling us there are evidently no magnetic monopoles on which to start and end magnetic field lines.

$\oint \vec{B} \cdot d\vec{l} = \mu_0(i_C + \varepsilon_0 \dfrac{d\Phi_E}{dt})$, telling us closed magnetic field lines are produced by the motion of charge and/or by changing electric flux.

$\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$, telling us closed electric field lines are evidently produced only by changing magnetic flux.

Suppose we have two coils, as in Fig. 30.1. A changing current in coil 1 will produce a changing magnetic field, giving changing magnetic flux through coil 2 and, by Faraday's law of induction, an induced emf in coil 2 : $\mathcal{E}_2 = -N_2\dfrac{d\Phi_{B2}}{dt}$, where $N_2\Phi_{B2} = M_{21}i_1$. Also $\mathcal{E}_1 = -N_1\dfrac{d\Phi_{B1}}{dt}$, where $N_1\Phi_{B1} = M_{12}i_2$. Advanced electromagnetism courses show $M_{21} = M_{12} = M$. Therefore, $\boxed{\mathcal{E}_2 = -M\dfrac{di_1}{dt}}$ and $\boxed{\mathcal{E}_1 = -M\dfrac{di_2}{dt}}$.

$\mathcal{E}_2$ is the emf (in V) induced in coil **2** due *solely* to the time rate of change of current $\dfrac{di_1}{dt}$ (in $\dfrac{\text{A}}{\text{s}}$) in coil **1**.

$\mathcal{E}_1$ is the emf (in V) induced in coil **1** due *solely* to the time rate of change of current $\dfrac{di_2}{dt}$ (in $\dfrac{\text{A}}{\text{s}}$) in coil **2**.

M is the **mutual inductance** (in henry = H).

Solving both $N_2\Phi_{B2} = Mi_1$ and $N_1\Phi_{B1} = Mi_2$ for M gives us $\boxed{M = \dfrac{N_2\Phi_{B2}}{i_1} = \dfrac{N_1\Phi_{B1}}{i_2}}$ .In this equation, all quantities are never negative.

N_2 and N_1 are the numbers of turns in coils 2 and 1 (no units).

Φ_{B2} is the average magnetic flux (in Wb) through each turn of coil **2** due *solely* to i_1, the current (in A) in coil **1**.

Φ_{B1} is the average magnetic flux (in Wb) through each turn of coil **1** due *solely* to i_2, the current (in A) in coil **2**.

The SI unit one henry is equivalently $1\ \text{H} = 1\ \dfrac{\text{Wb}}{\text{A}} = 1\ \dfrac{\text{T·m}^2}{\text{A}} = 1\ \dfrac{(\text{N/A·m})\text{·m}^2}{\text{A}} = 1\ \dfrac{\text{J}}{\text{A}^2} = 1\ \dfrac{\text{J}}{(\text{C/s})\text{·A}} = 1\ \dfrac{\text{V·s}}{\text{A}} = 1\ \Omega\text{·s}$.

Cover up the solutions and carefully work Examples 30.1 and 30.2.

Now suppose we consider a single coil, as in Fig. 30.4. A changing current in that coil will produce a changing magnetic field, giving changing magnetic flux through the coil and, by Faraday's law of induction, an self-induced emf $\mathcal{E}$ (in V): $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$, where $N\Phi_B = Li$, which gives $\boxed{\mathcal{E} = -L\dfrac{di}{dt}}$. Thus $\boxed{L = \dfrac{N\Phi_B}{i}}$ is the **self-inductance** L (in H) of that coil (often just called the *inductance* of that coil). In $L = \dfrac{N\Phi_B}{i}$, all four quantities are never negative.

Cover up the solutions and carefully work Examples 30.3 and 30.4.

Suppose we have two ordinary coils. Each coil will have a self-inductance (L_1 and L_2) and the two coils may also have a mutual inductance (M). Thus, for coil **1**, we can write $\mathcal{E}_{1,\text{self}} = -L_1\frac{di_1}{dt}$, with $L_1 = \frac{N_1(\Phi_B)_1}{i_1}$, where $(\Phi_B)_1$ is the average magnetic flux through each of the N_1 turns of coil **1** due *solely* to coil **1**'s current i_1. Thus $(\Phi_B)_1$ (from coil **1**'s current) is *not* the same average magnetic flux as Φ_{B1} (from coil **2**'s current). For coil **2**, we similarly write $\mathcal{E}_{2,\text{self}} = -L_2\frac{di_2}{dt}$, with $L_2 = \frac{N_2(\Phi_B)_2}{i_2}$ [where $(\Phi_B)_2$ (self) is *not* the same as Φ_{B2} (mutual)]. That is, we have three possible inductances (L_1,L_2, and M) and four possible emfs ($\mathcal{E}_{1,\text{self}}$, $\mathcal{E}_{1,\text{mutual}}$, $\mathcal{E}_{2,\text{self}}$, and $\mathcal{E}_{2,\text{mutual}}$).

An **inductor** (sometimes called a *choke*) is a circuit element used mainly for its inductance.

On page 999, the text derives an expression for the magnetic potential energy U (in J) stored in the magnetic field of an inductor of (self-)inductance L (in H) when carrying a current I (in A): $\boxed{U = \frac{1}{2}LI^2}$. The unit equivalence $1\ \text{H} = 1\ \frac{\text{J}}{\text{A}^2}$ is helpful in this equation.

Cover up the solution and carefully work Example 30.5.

Similar to using $U = \frac{1}{2}\ CV^2$ and a parallel-plate capacitor to find the energy density u of an electric field, we now use $U = \frac{1}{2}\,LI^2$ and a toroid to find the energy density u of a magnetic field. Consider a toroid of small cross section and two layers of wire wound so that its magnetic field is zero outside the "dough" of its doughnut-shaped core.

First we find its self-inductance $L = \frac{N\Phi_B}{i} = \frac{NBA}{i}$. We substitute $B = \frac{\mu NI}{2\pi r}$ and $i = I$ to find $L = \frac{\mu N^2 A}{2\pi r}$. By definition, $u = \frac{U}{\text{volume}} = \frac{\frac{1}{2}LI^2}{\text{volume}} = \frac{\frac{1}{2}\frac{\mu N^2 A}{2\pi r}I^2}{(2\pi r)A} = \frac{1}{2\mu}\left(\frac{\mu NI}{2\pi r}\right)^2 = \frac{1}{2\mu}\ B^2$. This final expression has none of the dimensions of the toroid left and is true for all linear materials (those with a constant μ): $\boxed{u = \frac{B^2}{2\mu}}$.

u is the energy density (in $\frac{\text{J}}{\text{m}^3}$) of the magnetic field $\vec{B}$ (in T).

μ (mu) is the **permeability** of the material (in $\frac{\text{T·m}}{\text{A}}$).

μ_0 is the permeability of vacuum ($\mu_0 \equiv 4\pi \times 10^{-7}\,\frac{\text{T·m}}{\text{A}}$).

Thus $\mu \equiv \mu_0$ by definition for vacuum and also for nonmagnetic materials. Because of their ordinarily weak magnetizations, μ is slightly greater than μ_0 for paramagnetic materials (if not at very low temperatures) and μ is slightly less than μ_0 for ordinary diamagnetic materials (not superconducting).

We *must* distinguish between U (magnetic potential energy), u (energy density), and μ (permeability). To minimize confusion, in this block we will *not* also use μ to stand for the magnitude of the magnetic dipole moment (as in $\mu = NIA$). However, we may use the metric prefix $\mu = 10^{-6}$, as in μ H.