

Consider a freely-pivoted compass needle (or a bar magnet) in the earth's *magnetic field*. The magnetized object's **north pole** (N-pole) points north and its **south pole** (S-pole) points south. Considering the earth as a magnet, currently its *north* magnetic pole is located near its *south* geographic pole and vice versa (see Fig.27.3).

$\vec{F}_0 = q_0\vec{E}$ defines the electric field \vec{E} and $\vec{F} = q\vec{v} \times \vec{B}$ defines the **magnetic field** \vec{B} . This vector equation tells us that any non-zero \vec{F} is perpendicular to both \vec{v} and \vec{B} in a right-hand sense, and that \vec{F} is in the same direction as $\vec{v} \times \vec{B}$ for positive charge and opposite $\vec{v} \times \vec{B}$ for negative charge (Fig. 27.8).

The magnitude of the magnetic force \vec{F} is $F = |q|vB \sin \phi = |q|v_{\perp}B = |q|vB_{\perp}$. Thus F is zero if the object moves parallel ($\phi = 0$) or antiparallel (opposite) ($\phi = 180^\circ$) to the external magnetic field (see Fig. 27.6a) and F is greatest when the charged object moves perpendicular ($\phi = 90^\circ$) to the external magnetic field (see Fig. 27.6c).

\vec{F} is the magnetic force (in N) on the object and q is the object's charge (in C, where $1 \text{ C} \equiv 1 \text{ A}\cdot\text{s}$).

\vec{v} is the object's velocity (in $\frac{\text{m}}{\text{s}}$) and \vec{B} is the external magnetic field (in T = tesla, where $1 \text{ T} = 1 \frac{\text{N}}{\text{A}\cdot\text{m}}$).

Cover up the solution and carefully work Example 27.1.

Magnetic field lines:

1. Used to visualize the magnetic field.
2. \vec{B} is tangent to a magnetic field line at any point.
3. B is larger where the lines are closer together (and smaller where farther apart).
4. Can be said to always form closed loops.

Recall that we define the *electric flux* Φ_E by $\Phi_E \equiv \int \vec{E} \cdot d\vec{A}$. Similarly, we define the **magnetic flux** Φ_B by $\Phi_B \equiv \int \vec{B} \cdot d\vec{A} = \int B \cos \phi dA = \int B_{\perp} dA$. If the magnetic field is uniform and the surface is flat, we can pull the constants out of the integrals to obtain $\Phi_B = \vec{B} \cdot \vec{A} = B \cos \phi A = B_{\perp} A$.

Φ_B is the magnetic flux (in Wb, where $1 \text{ Wb} = 1 \text{ weber} = 1 \text{ tesla}\cdot(\text{meter})^2 = 1 \text{ T}\cdot\text{m}^2$).

Cover up the solution and carefully work Example 27.2.

Gauss's law for magnetism is $\oint \vec{B} \cdot d\vec{A} = 0$. The net magnetic flux through any closed surface is zero because *there are evidently no N-poles or S-poles by themselves*, that is, **no magnetic monopoles**.

If the *only* interaction for a charged particle is the $q\vec{v} \times \vec{B}$ magnetic force, its speed remains constant.

On each free charge moving in a current-carrying straight wire in a uniform magnetic field, the average force is $\vec{F}_{\text{av}} = q\vec{v}_d \times \vec{B}$. As there are nAl free charges in a straight-wire segment, $\vec{F} = nAlq\vec{v}_d \times \vec{B}$ is the total

force. In your study of electrical circuits, you may find $(nAq\vec{v}_d)l = I\vec{l}$, so $\vec{F} = I\vec{l} \times \vec{B}$. The magnitude of \vec{F} is $F = IlB \sin \phi = IlB_{\perp} = Il_{\perp}B$. Thus the magnetic force is zero if the current flows parallel or antiparallel to the external magnetic field ($\phi = 0$ or 180°) and its magnitude is greatest when the current flows perpendicular to the external magnetic field ($\phi = 90^\circ$).

\vec{F} is the magnetic force (in N) on the wire: nonzero \vec{F} is always perpendicular to both \vec{l} and \vec{B} .

\vec{l} is the vector length (in m) of the straight-wire segment (with direction that the current flows).

\vec{B} is the uniform external magnetic field (in T) and I is the current (in A) in the wire.

Cover up the solution and carefully work Example 27.7.

If the wire is *not* straight and/or the magnetic field is *not* uniform, we can find the magnetic force by performing a vector integral of $d\vec{F} = I d\vec{l} \times \vec{B}$ as in Example 27.8.

A current loop forms a **magnetic dipole**. In a uniform external magnetic field, the net *force* on a magnetic dipole is zero, but there is a net *torque*. From Fig. 27.31a), $F = IaB \sin 90^\circ = IaB$. The magnitude of the torque on one turn is $\tau = Fr_{\perp} = (IaB)(b \sin \phi)$. For the rectangular current loop, $ab = A$ and, for a coil of N turns, $\tau = NIAB \sin \phi$. We define the **magnetic (dipole) moment** $\vec{\mu}$ by $\vec{\mu} \equiv NIA\vec{A}$. This vector equation shows us that $\vec{\mu}$ and \vec{A} always have the *same* direction. In magnitude, $\mu = NIA$, with the quantities never negative.

$\vec{\mu}$ is the magnetic dipole moment (in $A \cdot m^2$) and N is the number of turns (no unit).

I is the current (in A) and \vec{A} is the area vector (in m^2).

Another right-hand rule: Wrap the fingers of your right hand around the coil in the way the current flows. Your extended right thumb then points perpendicular to the plane of the coil in the direction of $\vec{\mu}$ and \vec{A} . For examples, see Figs. 27.31, 27.32, and other following figures. For a coil in the plane of the paper, you should be able to show that a clockwise current gives $\vec{\mu}$ and \vec{A} into the paper (\otimes) (away from the reader) and a counterclockwise current gives $\vec{\mu}$ and \vec{A} out of the paper (\odot) (toward the reader).

$\tau = NIAB \sin \phi$ and $\mu = NIA$ give us the magnitude of the torque: $\tau = \mu B \sin \phi$. In vector form,

$\vec{\tau} = \vec{\mu} \times \vec{B}$. Optional Sect. 27.8 explains how this torque spins the rotor of an electric motor.

$\vec{\tau}$ is the magnetic torque (in $N \cdot m$): a nonzero $\vec{\tau}$ is always perpendicular to both $\vec{\mu}$ and \vec{B} .

\vec{B} is the external magnetic field (in T).

ϕ is the angle between the directions of the two vectors $\vec{\mu}$ and \vec{B} ; $180^\circ \geq \phi \geq 0$.

In Block 1, $\vec{\tau} = \vec{p} \times \vec{E}$ gave $U = -pE \cos \phi = -\vec{p} \cdot \vec{E}$. Replacing \vec{p} with $\vec{\mu}$ and \vec{E} with \vec{B} , $\vec{\tau} =$

$\vec{\mu} \times \vec{B}$ gives $U = -\mu B \cos \phi = -\vec{\mu} \cdot \vec{B}$.

In this boxed equation, U is the **magnetic potential energy** (in J).

Cover up the solution and carefully work Examples 27.9 (which uses μ_{total} for what we call μ) and 27.10.

In the **Hall effect**, an electric field (and resulting potential difference) develop between the edges of a current-carrying slab in a transverse magnetic field (see Fig. 27.41).